Complex analysis

K1 Level Questions

UNIT I

1. In the complex number, Z=4=6i the real part is ……
   a. **4** b. 6 c. I d. none of the above
2. In the complex number Z= x+iy, the number is pure imaginary, If……….
   a. **X=0** b. y=0 c.i=0 d. none of the above
3. Two complex number Z_1= x+iy and z_2= a+ib are equal iff……………
   a. **x=a,y=b** b. x=b,y=a c.x=i,y=a d. x=b,y=i
4. For Z_1=4+3i the value of Re(Z_1^3) is ……
   a. 44 **b. -44** c. 64 d. -33
5. For Z=4+3i the value of $(ReZ)^3$ is……………….
   a. -64 b. 44 **c. 64** d. -44
6. If $\bar{Z}$ is the complex conjugate of z, then
   a. **Re Z= $\frac{}{2}$** b. Re Z= $\frac{1}{}$ $Z - \bar{Z}$ c. Re Z= $\frac{1}{}\bar{Z}$. d. Re Z= $\frac{1}{}Z$
7. Let Z=X+iY where $\bar{Z}$ is its complex conjugate, then for y=o,
   a. **Z- .** b.Z= $\frac{1}{}$. c.Z= -$\bar{Z}$ D. none of the above.
8. Let Z= x+iy, then its complex conjugate $\bar{Z}$ is ……
   a. $\bar{Z}$ xiy **b. x-iy** c. $\bar{Z}$= $\frac{1}{iy}$ d. none of the above
9. Let Z=x+iy, then im(Z)= .
   a. $\frac{y}{y}$ **b. $\frac{}{2}$+y²** c. $\frac{1}{y}$. d. $\frac{y}{y}$
10. If $\bar{Z}$ is the complex conjugate of Z, then…………..
    a. $\bar{Z}$ Z= Z^2. **b. Z= lZl²** c. –= lZl d. None of these.

UNIT II

1. in the polar co ordinate (r,θ) for the complex number Z=x+iy.
a. $\overline{\;2\quad 2\;}$. b. – c. $x + y = r$ $d.\ x - y = r$
2. Let Z_1=-2+2i Z_2= 2i, then Z_1Z_2 is equal to………
a. +6(1+i) **b. -6(1+i)** c. 6+i d. none of these
3. The nth root of unity are…
   a. $\sqrt{1}$= cos$\frac{\pi}{}$ - i sin$\frac{\pi}{}$, K=0,1… b. cos$\frac{\pi}{}$ i sin$\frac{\pi}{}$, **c. cos$\frac{2}{}$ + i sin$\frac{}{}$**, d. none of these
4. The polar representation of 1+i.
   a. $\sqrt{2}$ cos $\frac{\pi}{}$ **b. $\sqrt{2}$ (cos – +i –)** c. $\sqrt{2}$ cos π +isinπ d. $\sqrt{2}$ I sin π
5. The polar representation of ($\frac{i}{i}$) is…………
   a. 2 cos($\frac{\pi}{}$ isi $\frac{\pi}{}$) b. 4 cos($\frac{\pi}{}$ isi $\frac{\pi}{}$) c. 2 (s isi) **d. 4 ( )**
6. The point of the unit circlelZl forms.
   a. Open set b**. closed set.** c. semi open set. d. none of these

7. The complement of the unit circlelZl =1 is….
   a. open set. B. closed set. C. semiopen d. none of these.
8. An annulus ρ_1<lZ-αl < ρ_2 is ……
   a. **Connected** b. Dis connected. c. semi connected d. non of these
9. Half open plane is ……..
   a. **connected** b. Disconnected c. semi connected d. none of these
10. The complement of the unit circle lZl=1 is …
    a. **lZl>1** b.lZl<1 c. lZl=1 d. none of these

UNIT III

1. If a function f(Z) is continuous at $Z=Z_0$, the following statement does not hold…….
   a. **F is analytic at Z_0** b. f is defined at z_0 c. log $\rightarrow$ Z exist at z_0 d. none of these.
2. For the function $f(Z)=Z^2$ the value of derivative at Z=4 is ….
   a. 2 b. 5 c.**8** d. 6
3. The function f(Z) =lZl^2 is ..
   a. Every where analytic b**. No where analytic** c. analytic at Z=0 d. none of these
4. The function f(Z)= lZl^2 differentiable at …..
   a. **Z=0** b. $Z \neq 0$ c. nowhere d. none of these
5. The function f(Z) = Re (Z) is …….
   a. Analytic b. nowhere differentiable c**. continuous** d. none of these
6. The value of the derivative of f(Z)= $\frac{i}{i}$ at Z=i is……
   a. i/2 b. i/4 c**.-i/2** d. i/3
7. The value of the derivative of f(Z)= $\frac{i}{i}$ at any z is……
   a. **Zero** b. 6z c. z d. none of these
8. A function has isolated singularities at …………
   a. ∞ b. 0 c. 1/∞ d. none of the above
9. A function(1/z) have isolated singularities at ………
   a. ∞ b**. 0** c. 1/0 d. none of the above
10. A function F(Z)= Z^2 have zero of order……..
    a. One b**. two** c. three d. four.

UNIT IV

1. If f(Z) is analytic in a domain D, then…..
   a. **F^n(Z) exist in D** b. F^n(Z) does not exist in D c. F^n(Z) =0 for all n in D d. none of these.
2. A point Z= Z_0 is a singular point of analytic function f(Z), if ------------------
   a. At $z=z_0$, f(z) is not analytic
   b. At $z=z_0$, f(z) is analytic
   c. At $z=z_0$, f(z) =0
   d. None of these
3. The pole of the first order are known as
   a. Complex pole b**. simple pole** c.singularities d. none of these
4. The zero of the first order is known as….

a. Complex pole   b**. simple pole**   c.singularities   d. none of these

5. If f(z) is entire, then…..
   a. F(z) is analytic for all z
   b. F(z) is diverge for all Z
   c. F(z) is not analytic for all Z
   d. None of these
6. A: Every power series represents analytic function
   B: Every analytic function represent power series
   a. A is true, B is false
   b. B is true, A is False
   c. A and B both true
   d. None of these
7. A point Z_0 =0 is called Zero of f(z), if …..
   a. F(z_0) is constant   b**. f(z_0)=0**   c. f(z_0) $\geq$ 0   d. none of these
8. A Maclaurin series is a Taylor series with centre……..
   a. Z_0=1   b**. Z_0= 0**   c. Z_0 =2   d. none of these
9. A: every analytic function can be represented by power series
   B: Taylor series is a power series
   a. A and B both false
   b. A and B both true
   c. A is true and B is false
   d. A is false and B is true
10. If f(z) is entire function the Taylor series is………
    a. Convergent for all Z
    b. Divergent for all Z
    c. Constant
    d. None of these

UNIT V

1. If f and g are analytic function then………..
   a. f/g always analytic
   b. f/g is analytic whenever g(x)$\neq$ 0
   c. f/g is analytic whenever f(x)$\neq$ 0
   d. none of these
2. A function f(Z+c) =f(Z), where c is any number, then f is …………….
   a. A periodic function
   b. Periodic function with period c
   c. Periodic function with period z
   d. None of these
3. An analytic function is …..
   a. Infinitely differentiable
   b. Finitely differentiable
   c. Not differentiable
   d. None of these

4. If f is analytic and f'(z) ≠ 0, then ……..
   a. F is non conformal mapping
   b. F is a conformal mapping
   c. F is constant function
   d. None of these
5. For any point z_1,z_2,z_3,z_4 are distinct points and T is any Mobious transformation then the cross ratio (z_1,z_2,z_3,z_4) is equal to……..
   a. (Tz_1,Tz_2,z_3,z_4)
   b. (Tz_1,Tz_2,Tz_3,z_4)
   c. (Tz_1,Tz_2,Tz_3,Tz_4)
   d. None of these
6. The mobious transform takes…………..
   a. Circles in to line
   b. Circle into circle
   c. Circle into square
   d. None of these
7. If f is an entire function, then………..
   a. F has a power series expression
   b. F as not a power series expression
   c. F is constant
   d. None of these
8. If F is a bounded entire function, then……….
   a. **F is constant** b. f is equal to Zero c. f is increasing function d. f is decreasing function
9. A branch of logarithm function is
   a. Continuous function b. differentiable function c**. analytic function** d. none of these
10. If series $\sum$ converges absolutely, then…………..
    a. $\sum$ converges
    b. $\sum$ does not converges
    c. $\sum$ diverges
    d. None of these

Complex Analysis

K2 Level Questions

UNIT I

1. **Define Limit**

The function f(x) is said to have the limit A as X tends to a, $\lim_{x\to}$ =A, If and only if the following is true: for every ∈> 0 there exists a number >0 with the property that | − | < for all values of x such that | − |< and x ≠ a.

2. Define continuous function

A function f(x) is said to be continuous at a if and only if $\lim_{x\to}$ =f(a). A function f(x) is said to be continuous iff it is continuous at all point where it is defined.

3. Define Analytic function.

A complex function of a complex variable is said to be analytic if it is differentiable whenever it is defined.

4. Define conjugate harmonic function of U

If two harmonic functions u and v satisfy the Cauchy Riemann equations, then v is said to be the conjugate harmonic function of u.

5. Define Simple Zero.

If Zero of order one is called a simple Zero, if Z=α is a simple zero of P(x) then P(α)

=0 and $P^1(\alpha)\neq 0$.

6. State Lucas theorem

If all zeros of a polynomial p(z) lie on a half plane, then all the zeros of the derivative $p^1(z)$ lie in the same half plane.

7. Define Linear Transformation.

A rational function R(z) of order 1 is a linear function R(Z)=$\frac{\alpha Z}{Z\ \delta}$ with − ≠0, such fraction are called Linear transformation.

8.Define Parallel translation

The linear transformation w=z+a is called a parallel translation.

9. Define Inversion.

The linear transformation w=1/z is called a inversion.

10.Define Fixed point.

A plane z=α is said to be a fixed point of the transformation w=s(z) if s(α)=α.

UNIT II

1. **Define Sequence**
   A Sequence of Complex Numbers is an infinite ordered list of complex numbers, $(a_n)_{n=1}^{\infty}=(a_1,a_2,...,a_n,...)$, $a_n \in C$ for all $n \in N$
2. **Define Bounded Sequence**
   A complex sequence $\{z_n\}$ is bounded provided that there exists a positive real number R and an integer N such that $|z_n| < R$ for all $n > N$. In other words, for $n > N$, the sequence $\{z_n\}$ is contained in the disk $D_R(0)$.
3. **Define Cauchy Sequence.**
   The sequence $\{z_n\}$ is said to be a Cauchy sequence if for every $\epsilon > 0$ there exists a positive integer N_ϵ, such that if $n, m > N_\epsilon$, then $|z_n - z_m| < \epsilon$, or, equivalently, $z_n - z_m \in D_\epsilon(0)$.
4. **Define Infinite Series**
   The formal expression $\sum_{k=1}^{\infty} z_k = z_1 + z_2 + ... + z_n + ...$ is called an infinite series, and $z_1, z_2, ..., z_n, ...$, are called the terms of the series.
5. **When do a function Converge**
   A sequence of complex numbers $(a_n)_{n=1}^{\infty}$ is said to **Converge** to $A \in C$ if for all $\epsilon>0$ there exists an $N \in N$ such that if $n \geq N$ then $|a_n - A| < \epsilon$.
6. **When do a function Diverge**
   A sequence of complex numbers $(a_n)_{n=1}^{\infty}$ is said to **Diverge** if it does not converge to any $A \in C$.
7. **When a set will be open.?**
   A set is open iff it does not contain any boundary point.
8. **What is bounded?**
   A set is bounded iff it is contained inside a neighborhood of O.
9. **Define compact set**
   A set is compact iff it is closed and bounded.
10. **What is a disconnected set?**
    A set S is disconnected iff it is contained in the union of two disjoint, open sets A, B each of which contains at least one point of S.

UNIT III

1. **Define Exponential Function**

The exponential function is defined as the solution of the differential equation $f^1(z)=f(z)$with initial value $f(0)=1$ and it is denoted by e^z

2. **Define connected**

   A non empty open set in the plane is said to be connected if and only if any two of its points can be joined by a polygon which lies in the set.
3. **Define Region**

   A non empty connected open set is called a region
4. **Define Simply connected region**

   A region is said to be simpl connected if every closed curve in the region can be shrunk to a point without crossing the region.
5. **Define multiply connected region**

   A region which is not simply connected is called a multiply connected region
6. **Define open covering**

   A collection of open sets is an open covering of a set X if X is contained in the union of the sets.
7. **Define Compactness**

   A set X is Compact if and only if every open covering of X contains a finite subcovering
8. **Define Arc**

   A part of a curve is called a arc.
9. **Define Regular arc**

   A differentiable arc is said to be regular then the arc is said to be regular.

UNIT IV

1. **Define indefinite integral**

   Indefinite integral is a function whose derivatives equals a given analytic function in a region
2. **Define integral**

   The define integrals are taken ovr differentiable or piecewise differentiable arecs.
3. **State Cauchy Theorem for a rectangle.**

   If f(z) is analytic in a rectangle R then, $\int_{\delta R} \quad =0$
4. **State Cauchy theorem in a disk**

   If F(z) is analytic in an open disk Δ then $\int \quad =0$
5. **Define the Winding Number of a curve**

   The index of a point is the number of times a closed curve winds around the point.
6. **State Liouvilles theorem**

   A function wich is analytic and bounded in the whole plane must reduces to a constant.
7. **State Fundamental theorem of algebra**

   Every polynomial of degree n≠0 has atleast one zero.
8. **State moreras theorem**

If f(z) is defines and continuous in a region Ω and ∫ dz=0 for all closed curves , then f(z) is an analytic function in Ω.

9. State Chauchys integral formula.

Suppose f(z) is analytic on an open disk Δ. Let be closed curve on Δ., then for all in Δ with n(,z)=1

10. Define Holomorphic function

An analytic function is also called Holomorphic function.

UNIT V

1. **Define singularity**

   The point at which the function ceases to be analytic is called singularity

2. **Define Removable singularity**

   If z=a is a singularity of f(z) and if lim → exisits and finite, then z=a is called a removable singularity

3. **Define Zeros of a function**

   Zero of a function is a point at which the value of the function is zero

4. **Define Zero of order h**

   Z=a is said to be a zero of order h of the function f(z) if f(z)=(z-a)ϕ(z) where ϕ(z)≠0

5. **Define Isolated zero**

   If Z=a is a zero of f(z) then it is said to be isolated if a nbd about a in which f(z)≠0

6. **Define Isolated singularity**

   A singular point of a function is said to be isolated if there exists a nbd of that singularity point in which the function has no other singularity

7. **Define Pole**

   The point z=a is said to be a pole of function f(z) if lim →∞ ∞

8. **Define Meromorphic function**

   A function f(z) which is analytic in a region Ω, except for poles is called meromorphic function in the region.

9. **State weierstrass theorem on essential singularity**

   An analytic function comes arbitrarily closed to any complex value in every neighbourhood of an essential singularity.

10. **State Schwartz lemma.**

    If f(z) is analytic for | | 1 and satisfies the conditions | | 1, f(0)=0 then | | and | | 1

Complex Analysis

K3 Level Questions

UNIT I

1) If the F(x) is continuous then (i) Re f(x) (ii) Im f(x) (iii) |f(x)| are continuous.
2) A real function of a complex variable either has a derivate zero or else the deriviate does not exit
3) If f(z) is analytic show that |f(z) |2=J .,($\frac{u,}{,}$)
4) Show that the order of zero equals the order of the first non-vanishing deriviaties.
5) State the prove the Lucus theorem.
6) If f(z) and f(z) are analytical in a domain D such that f(z)=a constant in D
7) Show that u=1\2 log ^{2} ^{2} is harmonic
8) If f(z) is analytical, show that $\frac{df}{d}$=0
9) Show that f(z)=$\overline{z}$ is not analytic for any z
10) If R(z) =$\frac{P}{}$ has zero of order of ‘l’ then R’(z) has the same zeros as R(z), the order of each zero being reduced by one.

UNIT II

1. The real and imaginary part of Cauchy sequence is Cauchy sequence.
2. State and prove absolutely convergent.
3. The real and imaginary part of sequences, then original sequence converges.
4. Prove every convergent sequence is bounded.
5. State and prove Cauchy’s convergent test for the series.
6. The limit function is uniformly convergent sequence of continuous function is itself continuous.
7. Cauchy necessary and sufficient condition for uniform convergent.
8. Find the radius of convergent of following power series (i) ∑n (ii) $\sum \frac{}{!}$ (iii) n!z
9. A sequences is convergent if it is a Cauchy sequence
10. State and prove wirestrass M-test.

UNIT III

1. S and P addition theorem for exponential function
2. Show that, is never zero.
3. Show that >1 for x0 and 0< <1 for x<0
4. Show that exp$\overline{z}$ is complex conjugate of exp z.

5. Prove that sin Z = $\frac{iz \quad iz}{}$

6. Derive the Euler's formula and derive that $\cos^2 z+\sin^2$
7. Determine the formula for cosh2z+ sin2z.
8. Show that an arc is connected and compact.
9. Assume that argument of an anal.
10. If f (z) and f (z) are analytic in a region. Show that f (z) is constant.

UNIT IV

1. If c is complex constant then .
2. Line integral is invariant under a change of parameters.
3. Integral over a closed curve is invariant under a shift of parameters.
4. Compute z where is directed line segment from 0 to 1+i.
5. State and prove stronger theorem.
6. State and prove Cauchy's integral formula.
7. State and prove the integral formula'
8. If the integral z with continuous f depends only on the end points of then if the derivative of an analytic.
9. If the integral depends only on the end points of then the integral is an exact differential.

10. To prove ... z z+ z+ ….. + z

UNIT V

1. A non constant analytic function f(-Z) cannot obtain its maximum in a region Ω.
2. If f(z) is analytic on a closed set E and bounded set E then the maximum of |f(z) | is taken on the boundary of E.
3. State and prove maximum principal theorem.
4. Determine explicitly the largest disk above the origin whose image under the mapping $w=z^2+z$ is i-1
5. A non-constant analytic function maps open sets onto open sets.
6. If is a simple closed curve then — $\frac{f'}{f}$dz = no of zeros enclosed by .
7. State and prove Taylor's theorem.
8. State and prove Taylor's remainder theorem.

9. If f(z) is analytic in a region Ω containing a point ‘a’ such that f(a) and all other derivatives (a) vanish. Then f(z) id identically zero in Ω.
10. State and prove the local mapping.

Complex Analysis
K4 Level Questions

UNIT I

1) If f(x) and g(x) are continuous (i) f(x)=g(x) (ii) f(x)g(x) and (iii) $\frac{f\ x}{x\ ,}$ where g(x)≠ 0, are continuous .
2) Obtain necessary and sufficient condition, to be analytical in a region.
3) Explain about polynomials.
4) $\lim_{x\to}\ f\ x$ if (i)$\lim_{x\to}\ Ref\ x$ Re (iii) $\lim_{x\to}$ $f\ x$
5) $\lim_{x\to}\ f\ x$ ≪ ≫ $\lim_{x\to}\ f\ x$

UNIT II

1) State and prove abel's theorem.
2) State and prove Abel's limit theorem
3) Let $\{b_n\}$ be the contradiction of { } and if { is a cxauchy sequence. Then { is also a couchy sequence.
4) Show that a sequence { } is convergent if and only if it is a Cauchy sequence.
5) Find the radius of convergence of power series for (i)$\sum^{\infty}$ $\frac{\sqrt{\ }\ i}{i}$ (ii) $\sum$——.

UNIT III

1) Show that the power series so obtained converges in the whole plane.
2) State and prove the trigonometric function.
3) Show that $|\cos z|^2=\sin h^2y+\cos^2 -\cosh^2 y -\sin^2 z$.
4) If f(z) is analytic function in a region Ω and if f(z)≠ 0 in Ω, then the mapping w=f(z) is conformal in Ω.
5) If f(z) is analytic in the region Ω and if z= is a critical point in it such that f'(0. then the mapping is not conformal at z= .

UNIT IV

1) State and prove line integral as function of arc.
2) Necessary and sufficient condition for the integral p x to defined Only one end point.
3) State and prove Cauchy's theorem for a rectangle.
4) Cauchy's theorem in a disk (state and prove)
5) If the Piewise differentiable closed curve r does not pass through the point a, then prove that the value of the integral _is a multiple of 2πi

Unit V

1) State and prove Schwartz lemma.

2) State and prove Taylor's theorem.

3) Define Essential singularity and characterize it.

4) Show that the functions $w = e^z$ and $w = \sin z$ have essential singularities at ∞.

5) Prove that a non – constant analytic function maps open sets onto open sets.