Modern algebra

K1 Level Questions

UNIT I

1. If order of group G is p ,where p is prime then
   - **a. G is abelian**
   - b. G is not abelian
   - c. G is ring
   - d. None of these
2. If G is a group, for a ∈ G ,N(a) is the normalize of a then ∀ x ∈ N(a)
   - a. $\boldsymbol{xa}$ $\boldsymbol{ax}$
   - b. a e
   - c. a e
   - d. $a \neq a$
3. If G is a group , then for all a, b G
   - a. $(a$ $= a$
   - **b. $\boldsymbol{a}$ $\boldsymbol{a}$**
   - c. a a
   - d. a a
4. If G is a set of integers and $a.$ $\equiv$ $a -$, then G is
   - **a. Quasi-group**
   - b. Semi group
   - c. Monoid
   - d. Group
5. In a group G , for each element a ∈ G , there is
   - a. No inverse
   - **b. A unique inverse $\boldsymbol{a}$ ∈**
   - c. More than one inverse
   - d. None of these
6. If p is a prime number and $p\ |o$, en a ∈
   - **a. $\boldsymbol{a}$ ∈**
   - b. a ∉ G
   - c. a ⊂
   - d. a ⊃
7. If G is a group of order n then , order of identity element is
   - **a. One**
   - b. Greater than one
   - c. n
   - d. none of these
8. If a ∈ is of order n and p is prime to n , then order of a is
   - **a. n**

b. one
c. less than n
d. greater than n

9. If the order of elements a , a ∈ are $and\ n$ respectively then
   a. $\boldsymbol{m}$
   b. $\neq\ n$
   c. $n\quad 0$
   d. $none\ of\ ese$
10. If in a group , a ∈ ,the order of a is n and order of a is m then
   a. $\boldsymbol{m}\ \leq$
   b. $\geq\ n$
   c. n
   d. $none\ of\ ese$
11. The identity permutation is
   a. Even permutation
   b. Odd permutation
   c. Neither even nor odd
   d. None of these
12. The product of even permutation is
   a. Even permutation
   b. Odd permutation
   c. Neither even nor odd
   d. None of these
13. The inverse if an even permutation is
   a. Even permutation
   b. Odd permutation
   c. Even or odd permutation
   d. None of these
14. The product of $1\ 2\ 4\ 5\quad 3\ 2\ 1\ 5\ 4\ is$
   a.
   b. $1\ 5$
   c. $3\ 4\ 1$
   d. $1\ 5\ 3\ 1$
15. The inverse of an odd permutation is
   a. Even permutation
   b. Odd permutation
   c. Even or odd permutation
   d. None of these
16. If and are the inverse of some elements a ∈ , then
   a. $\boldsymbol{c}$

b. $\neq$
c. $, for\ sa\quad e$
d. $none\ of\ \ ese$

17. Let Z be a set of integers , then under ordinary multiplication(Z,·) is
    a. Monoid
    b. Semi group
    c. Quasi group
    d. Group
18. is a sub group of G if
    a.
    b. $\subset$
    c. $\supset$
    d. $\neq$
19. If G = $1, -1, i\ -\ i$ is a multiplicative group then order of $-i$ is
    a. One
    b. Two
    c. Three
    d. Four
20. If G is a group of even order ,∀ a≠ e if $a\quad e$ then G is
    a. Abelian group
    b. Sub group
    c. Normal group
    d. None of these

UNIT II

1. If G = $0, 1, 2, 3, 4$, the order of 2 is
    a. One
    b. Two
    c. Three
    d. Four
2. Every group of prime order is
    a. Cyclic
    b. Abelian
    c. Sub group
    d. Normal group
3. The number of elements in a group is
    a. Identity of group
    b. Order of group
    c. Inverse of group

d. None of these

4. A $one - one$ mapping of finite group onto itself is
   a. Isomorphism
   b. Homomorphism
   c. Automorphism
   d. None of these

5. If in a group , $\forall\ a \in$
   a. a a
   b. a a
   c. a a
   d. $None\ of\ \square ese$

6. If f 2 3 and 4 5 be two permutations on five symbols 1,2,3,4,5 $\square en$ $f$is
   a.

   b.

   c.

   d.

7. Given permutation is equivalent to
   a. (1 6 3 2)(2 1)
   b. (1 6 3 2)(1 1)
   c. (1 6 3 2)(4 5)
   d. (1 6 3 2)(5 4)

8. If the given permutation are , find
   a.

   b.

   c.

   d.

9. If number of left cosets of H in G are n and the number of right cosets of H in G are then
   a. $\boldsymbol{m}$
   b. $\geq n$
   c. $\leq n$

d. $None\ of$ ▯ese

10. If H is a sub group of finite group G and order of H and G are respectively $and\ n$ then
    a. $\boldsymbol{m}|$
    b. $n|$
    c. $\nmid n$
    d. $None\ of$ ▯ese

11. If is a finite group of order n ,then for every a∈ G , we have
    a. $\boldsymbol{a}$ **an identity element**
    b. a a
    c. a a
    d. $None\ of$ ▯ese
12. If and are two sub groups of G then following is also sub group of G
    a. ∩
    b. ∪
    c.
    d. $None\ of$ ▯ese
13. The set M of square matrices (of same order) with respect to matrix multiplication is
    a. Group
    b. Semi group
    c. Monoid
    d. Quasi group
14. If (G,∗) is a group and ∀ a , ∈ G $*a$ $*$ $*a$ e , then is
    a. Abelian group
    b. Non abelian group
    c. Ring
    d. Field
15. If G is a group such that a e , ∀ a ∈ ,then G is
    a. Abelian group
    b. Non abelian group
    c. Ring
    d. Field
16. If f 2 3 and 4 5 are two permutations on 1 , 2, 3, 4, 5 then f is
    a.

    b.

    c.

    d.

17. If n is the order of element a of group G then a e , an identity element if
   a. $|n$
   b. $|\boldsymbol{m}$
   c. n
   d. $n \nmid$

18. The order of element in a group G is
   a. One
   b. Zero
   c. Order of a group
   d. Less than order of a group

19. If a , a ∈ a group and order of a and a are and n respectively then
   a. n
   b. n
   c. $\boldsymbol{m}$
   d. *None of* ⍰*ese*

20. If a, ∈ , a group of order m then order of a and $a$are
   a. Same
   b. Equal to m
   c. Unequal
   d. None of these

UNIT III

1. If G = $1, -1$ is a group , then the order of 1 is
   a. One
   b. Two
   c. Zero
   d. None of these

2. The product of permutations $1\ 2\ 3 \cdot 2\ 4\ 3 \cdot 1\ 3\ 4$ is equal to
   a.

   b.

   c.

   d.

3. The permutations is equal to
   a. (1 5)(1 3)(2 4)

b. (1)(2)(3)
c. (1 3 5)(5 6)
d. (1 4 2)(5 3)

4. Given the permutation 1 2 3 4 5 6 7 then is
   a. (1 3 5 7 2 4)
   b. (1 4 7 3 6 2 5)
   c. (1 7 6 5 4 3 2)
   d. I

5. If 1 2 3 4 then is
   a. (1 3)(2 4)
   b. (1 3)
   c. (2 4)
   d. (2 3)(3 1)
6. Statement A : All cyclic group are abelian
   Statement B: The order of cyclic group is same as the order of its generator
   a. A and B are False
   b. A is true and B is false
   c. B is true , A is false
   d. A and B are true
7. Statement A : Every isomorphic image of a cyclic group is cyclic
   Statement B: Every homomorphic image of a cyclic group is cyclic
   a. Both A and Bare true
   b. Both A and B are false
   c. A is true only
   d. B is true only
8. A element a of an finite cyclic group G of order n is a generator of if 0 p n and also
   a. is prime to
   b. p is the multiple of n
   c. n is the multiple of n
   d. None of these
9. If is the finite group of order n, $a \in$ and order of a is , if is the cyclic then
   a. $\boldsymbol{m}$ n
   b. n
   c. n
   d. $None\ of\ these$
10. If $a \in$ is a generator of a cyclic group and order of a is n ∞ then order of a cyclic group m is
   a. $Infini$
   b. $\boldsymbol{m}$
   c. n

d. n

11. Let G = $1, -1$ then under ordinary multiplication (G , ·) is
   a. Monoid
   b. Semi group
   c. Quasi group
   d. Group

12. Let be a set of rational numbers under ordinary addition , is
   a. Monoid
   b. Semi group
   c. Quasi group
   d. Group
13. Let be a group of square matrices of same order with respect to matrix multiplication then It is not a
   a. Quasi group
   b. Abelian group
   c. Semi group
   d. None of these
14. If is a finite group , then for every $a \in$ then order a is
   a. Finite
   b. Infinite
   c. Zero
   d. None of these
15. In the additive group of integers , the order of every element $a \neq 0$ is
   a. Infinity
   b. One
   c. Zero
   d. None of these
16. In a additive group of integers , the order of identity element is
   a. Zero
   b. One
   c. Infinity
   d. None of these
17. In the additive group G of integers the order of inverse element a , $\forall\ a \in$ is
   a. Zero
   b. One
   c. Infinity
   d. None of these
18. The singleton 0 with binary operations addition and multiplication is ring and it is called

a. Zero ring
b. Division ring
c. Singleton ring
d. None of these

19. The element $a \neq 0 \in$, the commutative ring is an integral domain if
a. $a \quad 0, \in \quad and \quad 0$
b. $\boldsymbol{a} \quad , \in \quad \boldsymbol{a} \quad \boldsymbol{d} \quad \neq$
c. $a \neq o, \in \quad and \quad 0$
d. $a \quad 0 , \in \quad and \quad o$

20. A ring is a integral domain if
a. is a commutative ring
b. is a commutative ring with zero divisor
c. is a commutative ring with non zero divisor
d. is a ring with zero divisor

UNIT IV

1. A ring with binary operation addition is an abelian group. It with binary operation multiplication , $\forall a, \in , a. \quad . a$, then R is
a. Commutative ring
b. Integral domain
c. Field
d. Null ring

2. If a ring $\neq 0$ has a multiplicative identity 1 then
a. $1 \quad 0$
b. $1 \quad 0$
c. $\neq$
d. *None of* ese

3. An element $a \in \quad , \quad , \cdot$ a ring is nilpotent if for some positive integer n is
a. $\boldsymbol{a}$
b. $a \quad a$
c. $a \quad 1$
d. *None of* ese

4. The following statement is false
a. The intersection of two non- empty sub ring is a sub ring
b. The intersection of two non empty sub group is a sub group
c. A skew field have zero divisor
d. An integral domain have zero divisor

5. If f is a isomorphism of a ring (, , ·) onto ring (, , ·) and
: Isomorphic image of a field is a field

: Isomorphic image of a division ring is a division ring

: Isomorphic image of a ring with unity is a ring with unity

Then

a. , , ***a***

b. *and are fa se*

c. *and are fa se*

d. *is fa se*

6. A field having no proper sub field is
   - **a. Prime field**
   - b. Division field
   - c. Integral domain
   - d. None of these
7. The following statement is false
   - a. Every field is a integral domain
   - **b. Every integral domain is a field**
   - c. Every field is a ring
   - d. Every ring is a group
8. The non zero element a, of the ring , ,· are called zero divisors if
   - a. ***a***.
   - b. a. 1
   - c. a. ≠ 0
   - d. a. ≠ 1
9. A field is defined as
   - a. Division ring
   - b. Commutative ring
   - c. Integral domain
   - **d. Finite integral domain**
10. A commutative ring with unity is called integral domain if a, ∈
    - a. a 0 ⇒ $a \neq 0$, ≠ 0
    - **b.** ***a*** ⇒ ***a*** ,
    - c. a 0 ⇒ a
    - d. *None of ese*
11. The generators of the group = a, a , a , a e are

a. ***a***

b. *a and a*

c. *a and a*

d. *a and a*

12. If = 1, − 1 is a group with ordinary multiplication the order of − 1 is

a. One

b. Two

c. Zero

d. None of these

13. Let and $'$ be two arbitrary ring , ϕ : R→R' defined as ϕ(a) = 0 for all $a \in$ then ,

a. ϕ is homomorphism

b. ϕ is automorphism

c. ϕ is isomorphism

d. none of these

14. The homomorphism ϕ of rings in o' is isomorphism If the kernel I(ϕ) is

a. ϕ

b. I ϕ

c. I ϕ $'$

d. None of these

15. If U is an ideal ring , then

a. $/$ $\boldsymbol{a}$

b. $/$ $is\ a\ rin$

c. $is\ a\ rin$

d. $None\ of$?ese

16. A field is a

a. Vector space

b. Integral domain

c. Division ring

d. Commutative division ring

17. If integral domain D is of finite characteristic , then its characteristics is

a. Odd number

b. Even number

c. Prime number

d. Natural number

18. A commutative divisional ring is

a. Vector space

b. Group

c. Integral domain

d. Field

19. A commutative division ring is

a. Finite integral domain

b. Integral domain

c. Zero ring

d. None of these

20. If is a commutative ring with unit element , is a maximum ideal of if

a. $/$ $\boldsymbol{a}$ $\boldsymbol{d}$

b. $/$ $is\ a\ fie$ d

c. $is\ a\ fie$ d

d. $None\ of$?ese

UNIT V

1. If is a commutative ring with unit element , is a ideal of and / is a finite integral domain then

a. $\boldsymbol{max}$ $\boldsymbol{ma}$ $\boldsymbol{d}$ $\boldsymbol{a}$

b. $is\ no$ a a i $a idea$ of

c. $is\ a$ ini $a\ idea$ of

d. $None\ of$?ese

2. If R is an Euclidean ring and $a,$ $\in$ .If b ≠ 0 is not a unity R then

a. $\boldsymbol{d}$ $\boldsymbol{a}$ d $\boldsymbol{a}$

b. d a d a

c. d a d a

d. $None\ of$?ese

3. If is a commutative ring, with unit element then

a. Every maximal ideal is a prime ideal

b. Every prime ideal is a maximal ideal

c. Every ideal is a prime ideal

d. Every ideal is a maximal ideal

4. A set of all even integers is a ring

a. Commutative ring

b. Integral domain

c. Field

d. None of these

5. Every integral domain is not a

a. Field

b. Commutative ring

c. Ring

d. Abelian group with respective addition

6. If the ring R is such that (a =a , $a,$ $\in$, then

a. $\boldsymbol{c}$ $\boldsymbol{mm}$ $\boldsymbol{a}$

b. $is\ no$ o a i e

c. is $erorin$

d. $None\ of$?ese

7. If the ring is finite and commutative with unit element , then

a. Every maximal ideal is a prime ideal

b. Every ideal is a maximal ideal

c. Every prime ideal is a maximal ideal

d. a and c are both true

8. The set of square matrices of same order with respect matrix addition , is a

a. Quasi group

b. Semi group

c. Group

d. Abelian group

9. The set of square matrices order 2, with respect to matrix multiplication

a. Quasi group

b. Semi group

c. Monoid

d. Group

10. The set of all non singular matrices of same order with respect to matrix multiplication is

a. Quasi group

b. Monoid

c. Group

d. Abelian group

11. If $a, \quad \in$ is a group then b is the conjugate to a if exists c ∈ G

a. $\boldsymbol{c \quad ac}$

b. a

c. a

d. a

12. If N is a set of natural numbers then under binary operation a.b =a-b, (N, .)

a. Quasi group

b. Semi group

c. Monoid

d. Group

13. If $e \ and \ e$ are two identity element of a group G then

a.

b. $e \neq e$

c. $e \quad e, for\ so \quad e$

d. $None\ of$?ese

14. is a set of even integers under ordinary addition and multiplication , then is a ring , is also

a. Commutative ring

b. Integral domain

c. Field

d. None of these

15. If the ring has left identity $e \ and\ ri$? $iden \quad i \ e$ then

a.

b. $e \neq e$

c. $e \quad e$

d. $none\ of$?ese

16. If the ring has a unity $e \ and\ e$?en

a.

b. $e \neq e$

c. $e \quad e$

d. *none of these*

17. If in a ring with unity (, ∀ , ∈) then

a. *is a o arini e*

b. ***a d ma***

c. *is a fie d*

d. *None of these*

18. If I is a integral domain and $a \neq 0 \in I$ then

a. $a \quad 0$

b. $a \geq 0$

c. $\boldsymbol{a} \neq$

d. None of these

19. If integral domain I is of finite characteristic then

a. *I is fini on*

b. *I is infini on*

c.

d. *None of these*

20. A ring (, + , ·) Is said to have zero divisor if

a. $a, \in , a. \quad 0 \Rightarrow a \neq 0$ *or* $\neq 0$

b. $\boldsymbol{a}, \in , \boldsymbol{a}. \Rightarrow \boldsymbol{a} \neq$ ***a d*** $\neq$

c. $a, \in , a. \quad 0 \Rightarrow a \quad 0$ *or* 0

d. $a, \in , a. \quad 0 \Rightarrow a \quad 0$ *and* 0

Modern Algebra

K2 Level Questions

UNIT I

1. **Define Sub Group.**
   A Subset H of group G is called a subgroup of G if H forms a group with respect to the binary operation in G.
2. **Define Group.**
   A Non empty set of element G is said to form a group, if in G there is defined a binary operation with Closure, Associative, Identity, inverse Property under addition or multiplication is a group.
3. **Define Closure Property under addition.**
   $a+b \in G$ for every a and b belongs to G.
4. **Define Associate Property under addition.**
   $(a+b)+c = a+(b+c)$ where $a,b,c \in G$
5. **Define identity property under addition.**
   $a+e=e+a=a$, e is the identity element for all $a \in G$
6. **Define Inverse property under addition.**
   $a+(-a)=(-a)+a=0$, -a is inverse of a where $a \in G$
7. **Define Abelian group.**
   A group is said to be abelian commutative group, if it satisfies the condition a.b=b.a for all a,b belongs to G
8. **Define Order of G**
   Number of elements in a group is called as order of G and it is denoted by O(G).
9. **Define Symmetric Group of degree n.**
   If a group has n elements then it is denoted as Sn and it is called as Symmetric Group of degree n.
10. **Define Generator of G**
    Let G be a group and let a belongs to G if <a>=G

UNIT II

1. **Define Cyclic Group.**
   A group G is cyclic if there exists an element a belongs to G such that <a>=G
2. **Define Cyclic Sub group.**
   Let G be a group. Let a belongs to G. Then $H=\{a^n/n \in Z\}$ is a subgroup of G. H is called the cyclic subgroup of G generated by a and it is denoted by <a>
3. **Define Center of a group.**
   If G is a group, then the center of G, denoted by z(G), is given by $Z(G)=\{a \in G/ab=ba \text{ for all } b \in G\}$.

4. **Define Coset:**
   Let H be a SubGroup of a group G. Let a ∈ G then the set aH{ah/h∈ H} is called the left coset of H defined by a in G.
5. **Define Index Of H in G**
   Let H be a subgroup of G. The number of distinct left coset (or) right cosets of H in G is called the index of H in G and is denoted by [G: H]
6. **State Lagrange theorem.**
   If G is a finite Group and H is a Subgroup of G, then o(H) is a divisor of O(G).
7. **Define Order of a.**
   If G is a group and a ∈ G, the order of a is the least positive integer m Such that a^m=e.
8. **State Euler's Theorem**
   If n is a positive integer and a is relatively prime to n, then $a^{\phi} \equiv 1$ mod n.
9. **State Fermat theorem:**
   If P is Prime Number and a is any integer, then $a^P \equiv$ a mod P.
10. **Define normal Sub Group.**
    A sub Group N of G is called a Normal Subgroup of G if aN=Na all a ∈ G.

UNIT III

1. **Define Quotient Structure.**
   Let G/N denote the collection of right cosets of N in G. Elements of G/N is the subsets in G.
2. **Define Quotient Group.**
   If G is a group, N a normal subgroup of G, then G/n is also a group. It os called the quotient group or factor group of G by N.
3. **Define Homomorphism.**
   A Mapping ϕ from a group G into a group G is said to be homomorphism, If for all a,b ∈ G, ϕ**(ab)=** ϕ ϕ **.**
4. **Give an Example for Homomorphism.**
   If (G,+) and (G,∗) be a group then f(x)= 2^x is Homomorphism of G
5. **Define Kernel of ϕ.**
   If ϕ is a homomorphism of G into G, the kernel of ϕ, K ϕ, is defined by k ϕ = { x ∈ G / ϕ (x)= $\bar{e}$, $\bar{e}$ = identity element of }.
6. **Define Isomorphism**
   A Homomorphism ϕ from G into G is said to be an isomorphism if ϕ is one to one
7. **Give an example for isomorphism.**
   If (G,+) and (G,∗) be a group then f(x)= 2^x is Isomorphism of G
8. **State Fundamental theorem for Homomorphism.**
   Let ϕ be a homomorphism of G into G with kernel K. Then G/K ≈ G.
9. **Define a Non trivial normal Sub group.**

A Group is said to be simple if it has no non trivial homomorphic images, that is if it has no non trivial normal subgroup.

10. **State Cauchys theorem for abelian Group**

Suppose G is a finite abelian Group and P/o(g), where P is a prime number. Then there is an element $a \neq e \in G$ such that $a^p=e$.

UNIT IV

1. **State Sylows theorem for abelian group.**

   If G is an abelian group of order o(G) and if P is a prime number, such that $P^{\alpha}/o(G)$. $P^{\alpha+1}$ does not divides o(G), then G has a Subgroup of Order P^{α}.

2. **Define inner automorphism.**

   If G be a group for $g \in G$ define the mapping $T_g: G \to G$ by $T_g(x) = gxg^{-1}$ for all $x \in G$. Then Tg is an automorphism of G, called the inner automorphism of G.

3. **Define Group of inner automorphism.**

   I(G) is a subgroup of A(G). I(G) is called the group of inner automorphisms.

4. **State Cayleys theorem**

   Every group is isomorphic to a subgroup of A(s) for some appropriate S.

5. **Define Transposition**

   A Permutation of cycle of length two is called Transposition.

6. **Define Even Permutation.**

   A permutation $\theta \in S_n$ is said to be an even permutation if it can be expressed as product of an even number of transpositions.

7. **Define Ring with unit element.**

   If there exist an element in R Such that a.1=1.a=a for every a in R. then R is called a ring with unit element

8. **Define commutative ring.**

   If a.b=b.a in R then R is called Commutative ring.

9. **Give an example for commutative ring with unit element.**

   R is the set of integer +ve , -ve and 0. +ve is the usual addition and (.) is the usual multiplication of integer and is a commutative ring with unit element

10. **Give an example for commutative ring but has no unit element**

    R is set of even integers under the usual operation of addition and multiplication. Thus R is commutative ring but has no unit element.

UNIT V

1. **Define Zero Divisor**

   If R is a commutative ring then $a \neq 0 \in R$ is said to be a zero divisor, if there exist $a,b \in R$, such that a.b =0

2. **Define integral domain**

   A commutative ring is an integral domain if it has no zero divisor

3. **Define Zero divisor**

   A ring is said to be a division ring if its non zero elements forma group under multiplication.

4. **Define Field**
   A commutative divisor ring is called a field
5. **Define Finite Field**
   If field has only finite number of elements then it is called finite field
6. **State Pigen hole principle**
   If n objects are distributed over m places and if n> m, then some plces receives atleast two object.
7. **Define characteristic 0**
   An intergral domain D is said to be of characteristic 0, if the relation ma=0, when a≠0 is in D and where m is an integer can hold only if m=0.
8. **Define Finite Characteristic.**
   An intergral domain D is said to be finite characteristic if there exists a +ve integer m . Such that ma=0 for all a ∈ D
9. **Define Homomorphism.**
   A mapping ϕ from the ring R into the Ring ^{1} is said to be homomorphism if
   ϕ(a+b)= ϕ a ϕ .
   ϕ(ab)= ϕ a ϕ .
10. **Define Zero Homomorphism**
    Trivially ϕ is a homomorphism and I(ϕ). ϕ is called Zero Homomorphism.

Modern Algebra

K3 Level Questions

UNIT I

1. State and Prove $cancellation\ la$.
2. Prove that if G is an abelian group, then for all $a,\ \in G$ and all integers $n,\ a.\ \ a$ ·
3. If H is a non empty finite sub set of a group $G\ an\ \ H$ is closed under multiplication, and then H is a sub group of G.
4. For all $a \in G, Ha\ \ \in - \equiv\ mo\ \ H$
5. State and Prove $Euler's\ t\text{?}eorem.$
6. State and Prove $Fermat's\ t\text{?}eorem.$
7. Given $a,$ in a group G , then the equation $a.\ \ an\ \ .a$ have a unique solution for $an\ \ in\ G.$
8. If G is a group in which $a.\ \ a$. for three consecutive integers $o\pi all\ a, \in G$, show that G is abelian.
9. A non empty subset H of the group G is a sub group of G if
   I. $a,\ \in H\ implies\ t\text{?}at\ a\ \ \in H$
   II. $a \in H\ implies\ t\text{?}at\ a\ \ \in H$
10. Define sub group and if H is a non empty finite sub set of a group $G\ an\ \ H$ is closed in a multiplication, then H is a sub group of G.

UNIT II

1) HK is a subgroup of G if HK is a subgroup of G if and only if HK=KH.
2) The subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G
3) If H and K are finite subgroup of G of orders O(H) and O(K) respectively, then

   O (HK)= $\frac{}{\cap}$
4) If H and K are subgroup of G and O(H)> $\overline{\ \ G}$, O(K)> $\overline{\ \ G}$ then, H∩K ≠(e)
5) If G is a group, N a normal subgroup of G, then – is also a group.
6) Show that every subgroup of an abelian group is normal.
7) If Φ is a homomorphism of G into Ḡ, then
   1. Φ(e) = ē, the unit element of Ḡ
   2. $\Phi(x^{-1})=\Phi(x)^{-1}$ for all X€G
8) If Φ is a homomorphism of G into Ḡ with Kernel K, then K is a normal subgroup of G.
9) Fundamental homomorphism of theorem.
10) Cauchy's theorem

UNIT III

1) Every permutation is a product of a cycle.
2) Every permutation can be uniquely repressed as a product of disjoint cycles.

3) Every permutation is a product of 2 – cycle.
4) If T: S→S , x € S, x € S, then x I is the image of x under I.
5) If G is a group then A(G), the set of of automorphism of G, is also a group.
6) I(G)≈– where I(G) ia a group inner automorphism of G and Z, is center of G.
7) Let G be a group and of an automorphism of G if a € G is of order O(G) >0 then O(Φ(a)) =O(a).
8) State and prove cayley's theorem.
9) If G is a group, H is a subgroup of \G and S. The set of all right cosets of H in G, there is a homomorphism O of G into A(S) and the Kernel of O is the largest normal subgroup of G which is contained in H.
10) Define : Inner automorphism, automorphism.

UNIT IV

1. If R is a ring then for all a,b€ R.
    - a. 0 = 0.a = 0
    - a(-b)= (-a)b = -(ab)
    - (-a)(-b)= ab

    If in addition and has a unit element, Then

    - (-1)a = -a,
    - (-1)(-1)=1
2. A finite integral domain is a field.
3. If Φ is a homomorphism of R into R', then
    - Φ(0) = 0
    - Φ(-a) = -Φ(a) ∀ a € R.
4. If Φ is a homomorphism of R into R' with Kernel I(Φ)then
    - I(Φ) is a subgroup of R under ' +'
    - If a € I(Φ) r € R then both ratar are in I (Φ).
5. The homomorphism Φ of R into R' is an isomorphism if and only if I(Φ) = 0.
6. If U is a ideal of the ring R then– a ring and is a homomorphism image of R.
7. Let R,R' be rings and Φ is homomorphism of R into R' with Kernel U . Then R' in isomorphic to – more over there is a one – one correspondence between the set of ideal of R' and set of ideal of R which contained O. This correspondence can be achieved by associating with an ideal 'U' in 'R' the ideal w in F defined by W= { x€ R}/ Φ(x) € O'}, If W defined — in a isomorphic to —
8. If R is a commutative ring with unit element and M is an ideal of R. Then M is maximum ideal of R if — is a field.
9. Every interval domain can be imbedded in a field.
10. Let R be the Euclidean ring and let 'A' be an ideal of R. Then exists an element a_o€ A such that A consist exactly of all a_ox where x rang over R.

UNIT V

1) Let R be a commutative ring with unit element whose only ideals are (O) and R itself. Then R is a field.
2) If R is a commutative ring with unit element and M is an ideal of R

   M is maximum ideal of R if — is a field.

3) Every integral domain can be imbedded in a field.
4) Explain – Euclidean ring.
5) Let R be the Euclidean ring and Let ‘A’ be an ideal of R Then exists an element a_0 € A such that A consists exactly of all a_0 x where x ranges over R.
6) A Euclidean ring possess a unit element.
7) Let R be a Euclidean ring the any two elements a and b in R have greatest common divisor d more over d= λa+µb for some λ,µ € R.
8) Let R be an integral domain with unit element and suppose that for a , b € R both – and

   – are true. Then a=ub , where u is a unit in R.
9) A element a in a equident ring FR is a unit if and only if d(a) = d(1).
10) Let R be an equident ring then every element in R is either a unit in R (or) can be written as the product of a finite number of prime element of R.

Modern Algebra

K4 Level Questions

UNIT I

1. There is a one-to-one corresponding between any two right co-sets of H $in\ G$.
2. If G is a group , then
   a. $e\ identity\ element\ of\ G\ i\ \ unique$
   b. $Every\ \ \in G\ \ \ unique\ inver\ \ in\ G$
   c. $for\ every\ \ \in G,$
   d. $for\ \ ll\ ,\ \ \in G,\ \ .\ \ \ \ \ \ .$
3. The relation $\equiv\ \ mod$ is an equivalence relation and define cyclic sub group.
4. State and prove $l\ \ \ r\ \ ret\ eorem.$
5. Let G be the set of all real 2 2 matrices $\left[\ \ d\right.$ where $d-\ \ \neq 0$ is a rational number.

   Prove that G forms a group under matrix multiplication.

UNIT II

1) Verify the following three facts.
   1. G≈G
   2. G≈G* implies G*≈G
   3. G≈G*, G*≈G** implies G≈G**
2) Given an example of a group G, subgroup H, and an element a€G. such that aHa^{-1}CH but aHa^{-1}≠H.
3) Show that the intersection of two normal subgroup of G is a normal subgroup of G.
4) If G is a finite group and N is a normal subgroup of G, then O(–)=——.
5) If Φ is a homomorphism of G into Ḡ with Kernel K, then K is a normal subgroup of G.

UNIT III

1) Every group is isomorphism to a subgroup of A(S) for some appropriate S.
2) Prove : O(Φ(a))= m= O(a).
3) Every permutation is a product of 2 – cycle.
4) If T: S→S , x € S, x € S, then x I is the image of x under I.
5) If G is a group then A(G), the set of ofautomorphism of G, is also a group.

UNIT IV

1. Let R be a Euclidean ring then any two elements a and b in R have greatest common divisor d more over d= λa + µb for some λ,µ € R.
2. Let R be a Euclidean ring then every element in R is either a unit in R or can be written as the product of a finite number of prime elements of R.
3. Let R be a Euclidean ring suppose that for a,b,c € R , $\frac{}{c}$ but (a,b) = 1. Then $\frac{}{c}$.
4. J[i] is aneuclidea ring.

5. If U is a ideal of the ring R then– a ring and is a homomorphism image of R.

UNIT V

1) Let R be equident ring suppose that for abc € R — $_c$ but (a.b) =1 then $\frac{}{c}$.
2) If π is a prime element in the equident ring R and — where ab €R. Then π divides atleast one of a or b.
3) State and prove unique factorization theorem.
4) The ideal A = a_0 is a maximum ideal of the equident ring R. if a_0 is a prime element R.
5) J[i] is an Euclidean ring .