

# NUMERICAL TECHNIQUES

## K1 Level Questions

### UNIT I

1.. The Newton Raphson algorithm for finding the iterative formula of reciprocal of N is

- A.  $x_{n+1} = x_n (2 - Nx_n)$
- B.  $x_{n+1} = x_n (2 + Nx_n)$
- C.  $x_{n+1} = x_n (4 + Nx_n)$
- D.  $x_{n+1} = x_n (4 - Nx_n)$

2. The another name of bisection method

- A. **Bolzano's method**
- B. Newton's method
- C. Method of falsi position
- D. Iteration method

3. A better and closer approximation to the root can be found by using an iterative process called

- A. **Newton's Raphson method**
- B. Bisection method
- C. Regula falsi method
- D. Iteration method

4. The linear equation is two solutions is

- A. **Two solution**
- B. Three variables
- C. No variable
- D. One variable

5. The convergence of which of the following method is sensitive to starting value?

- A. Regula falsi method
- B. **Newton Raphson method**
- C. Gauss seidal method

D. None of these

6. Newton-Raphson method is used to find the root of the equation  $x^2 - 2 = 0$ . If iterations are started from -1, then iterations will be

A. Converge to -1

B. Converge to  $\sqrt{2}$

**C. Converge to  $-\sqrt{2}$**

D. No Converge

7. Double (Repeated) root of  $4x^3 - 8x^2 - 3x + 9 = 0$  by Newton-Raphson method is

A. 1.4

**B. 1.5**

C. 1.6

D. 1.55

8. Newton –Raphson method is applicable to the solution of

**A. Both algebraic and transcendental equation**

B. Both algebraic and transcendental equation and also used when the roots are complex.

C. Algebraic only

D. Transcendental equation only

9. Which one of the following functions is transcendental?

**A.  $a + be^x + c \sin x$**

B.  $x^2 - 5x + 6$

C.  $x^2 + 4$

D.  $3x^2 + 7x$

10. Order of convergence of Newton- Raphson method is \_\_\_\_\_

A. 1

**B. 2**

C. 3

D. 5

## UNIT II

1.  $(1 + \Delta)(1 - \nabla) = \underline{\hspace{2cm}}$   
A. 2  
**B. 1**  
C.  $\delta^2$   
D.  $\Delta$
2.  $\Delta$  is called  $\underline{\hspace{2cm}}$  operator.  
A. Shift  
B. Central difference  
**C. Forward difference**  
D. Backward difference
3.  $hD = \underline{\hspace{2cm}}$   
A.  $\log(1 - \Delta)$   
**B.  $\log(1 + \Delta)$**   
C.  $-\log(1 + \Delta)$   
D.  $-\log(1 + \nabla)$
4. The first difference of a constant is  $\underline{\hspace{2cm}}$   
A. 1  
B. 3  
C. 2  
**D. 0**
5. If  $\Delta f(x) = f(x+h) - f(x)$ , then a constant  $k$ ,  $\Delta k$  equals  
A. 1  
**B. 0**  
C.  $f(k) - f(0)$   
D.  $f(x+k) - f(x)$
6. Let  $h$  be the finite difference, then forward difference operator is defined by  $\underline{\hspace{2cm}}$ .  
A.  $f(x) = f(x-h)$   
B.  $f(x) = f(x-h) - f(x)$   
C.  $f(x) = f(x+h)$   
**D.  $f(x) = f(x+h) - f(x)$**
7. The value of  $E\nabla = \underline{\hspace{2cm}}$   
A.  $E$

**B.  $\Delta$**

C.  $\nabla$

D. 1

8. The displacement operator is also known as \_\_\_\_\_.

**A. Shift operator**

B. Mean operator

C. Central difference operator

D. Difference operator

9. Say True or False:  $1 + \Delta = (1 - \nabla)^{-1}$

**A. True**

B. False

10. To find the derivative at the end of the table we use \_\_\_\_\_ formula.

A. Newton's forward interpolation formula

**B. Newton's backward interpolation formula**

C. Lagrange's interpolation formula

D. Inverse Lagrange's interpolation formula

### UNIT III

1.  $E =$  \_\_\_\_\_

A.  $1 - \nabla$

B.  $1 + \Delta$

C.  $(1 - \nabla)^{-1}$

D.  $(1 + \nabla)^{-1}$

2.  $(1 + \Delta)(1 - \nabla) =$  \_\_\_\_\_

A. 2

**B. 1**

C.  $\delta^2$

D.  $\Delta$

3. The Symbol of Forward Operator is

A.  $\Delta$

**B.  $\delta^2$**

C.  $1 - \nabla$

D. None

4. If interpolation is required for the range  $\frac{-1}{2} < u < \frac{1}{2}$  then we use \_\_\_\_\_

- A. **Bessel's formula**
- B. Gauss backward formula
- C. Gauss forward formula
- D. Stirling's formula

5. In Newton's forward interpolation formula  $u =$  \_\_\_\_\_

- A.  $\frac{x - x_0}{h}$
- B.  $\frac{x + x_0}{h}$
- C.  $\frac{x - x_n}{h}$
- D.  $\frac{x + x_n}{h}$

Answer: A

6. In Newton backward interpolation formula  $u =$  \_\_\_\_\_

- A.  $\frac{x - x_0}{h}$
- B.  $\frac{x + x_0}{h}$
- C.  $\frac{x - x_n}{h}$
- D.  $\frac{x + x_n}{h}$

Answer: C

7. Interpolation means

- A. **Adding new data points**
- B. Only aligning old data points
- C. Only removing old data points
- D. None of the mentioned

8. Interpolation provides a mean for estimating functions

- A. At the beginning points
- B. At the ending points
- C. **At the intermediate points**
- D. None of the mentioned

9. Relation between  $\Delta$  and  $E$  is \_\_\_\_\_

- A.  $E = 1 + \Delta$
- B.  $E = 1 - \Delta$
- C.  $E = 1 + \nabla$

D.  $E = 1 - \nabla$

Answer: A

#### UNIT IV

1. In Simpson's 3/8 rule,  $y(x)$  is polynomial of degree

A. 1

B. 2

**C. 3**

D. 4

2. Runge-Kutta method is better than Taylor's method because

**A. it does not require prior calculations of higher derivatives**

B. it requires at most first order derivatives

C. it requires prior calculations of higher derivatives

D. all the above

3. The highest order of polynomial integrand for which Simpson's 1/3 rule of integration is exact is

A. first

B. second

**C. third**

D. fourth

4. When does Simpson's rule give exact result

A. if the entire curve  $y = f(x)$  is itself a hyperbola.

**B. if the entire curve  $y = f(x)$  is itself a parabola.**

C. if the entire curve  $y = f(x)$  is itself an ellipse.

D. if the entire curve  $y = f(x)$  is itself a circle.

5. What is the order of error in trapezoidal formula?

A.  $h$

B.  $h^3$

**C.  $h^2$**

D.  $h^4$

6. Taylor series method will be very useful to give some \_\_\_\_\_ for powerful numerical methods.

A. Initial value

B. Final value

C. Initial starting value

**D. Middle value**

7.  $y_{n+1} = y_n + h f(x_n, y_n)$  is the iterative formula for

- A. Euler's method**
- B. Taylor's method
- C. Adam's method
- D. Milne's method

8. Which of the following method is called step by step method

- A. Taylor's method
- B. RK method**
- C. Milne's method
- D. Newton's method

9. Simpsons one-third rule will give exact result, if the entire curve  $y = f(x)$  is itself a \_\_\_\_\_

- A. Ellipse
- B. Circle
- C. Parabola**
- D. Hyperbola

10.  $\int_a^b y dx = \frac{h}{2}(A + 2B)$  where A is \_\_\_\_\_

**A. Sum of the even ordinates**

B. Sum of the odd ordinates

C. Sum of the all ordinates

D. All these above

**UNIT V**

$$\int_a^b y dx = \frac{h}{3} [y_1 + 4(y_2 + y_4 + \dots + y_{2n}) + 2(y_3 + y_5 + \dots + y_{2n-1}) + y_{2n+1}]$$

1. \_\_\_\_\_ is

A. Trapezoidal rule

**B. Simpson's  $\frac{1}{3}$  rd rule**

C. Romberg rule

D. Simpson's 3/8 th rule

2. Whenever Trapezoidal rule is applicable, Simpson rule can be applied.

A. True

**B. False**

3.  $\frac{1}{h^2}[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12} \nabla^4 y_n + \dots]$  = \_\_\_\_\_

A.  $\left(\frac{dy}{dx}\right)_{x=x_0}$

B.  $\left(\frac{d^2y}{dx^2}\right)_{x=x_0}$

C.  $\left(\frac{dy}{dx}\right)_{x=x_n}$

**D.  $\left(\frac{d^2y}{dx^2}\right)_{x=x_n}$**

ANSWER : D

4. By Trapezoidal rule  $\int_0^6 \frac{1}{1+x} dx$  by dividing the range into six equal parts is=

A.1.9459

B.1.9587

C.1.9666

**D.2.0214**

5. In Euler's method, if h is small, the method is too slow and if h is large, it gives ----- value.

**(a) Inaccurate**

(b) Accurate

(c) Indefinable

(d) Zero

6. The improved Euler method is based on the average of-----

**(a) Slopes**

(b) Points

(c) Curve

(d) None of these

7. The modified Euler method is based on the average of-----

(a) Curve

- (b) Slopes
  - (c) Points**
  - (d) All the above
8. Runge-kutta formulas involve the computation of  $f(x,y)$  at various points instead of calculation of-----order derivatives of  $f(x,y)$
- (a) Lower
  - (b) Higher**
  - (c) Middle
  - (d) Neither (a) or (b)
9. Euler's modified formula is a particular case of -----order Runge-kutta method.
- (a) Third
  - (b) First
  - (c) Fourth
  - (d) Second**
10. The numerical solution of a first order differential equation will give a result is
- (a) A set of tabulated values of  $x$  and  $y$
  - (b) Value of  $x$  and  $y$**
  - (c) Zero
  - (d) Both (a) and (b)

## Numerical Techniques

### K2 Level Questions

#### Unit 1

1. Using Newton-Raphson method, find a root correct to three decimal places of the equation  $\sin x = 1 - x$ .

Answer: **0.511**

2. Using Bisection method, negative root of  $x^3 - 4x + 9 = 0$  correct to three decimal places is

Answer: **-2.706**

3. Newton-Raphson method of solution of numerical equation is not preferred when

Answer: **The graph of  $f(x)$  is nearly horizontal where it crosses the x-axis**

4. Which of the following statements applies to the bisection method used for finding roots of functions?

Answer: **Guaranteed to work for all continuous functions**

5. Which of the following method gives the comparatively faster conversion?

Answer: **Newton Raphson method**

6. In bisection method while taking the initial value for the two points which of the following condition

Answer: **Both must give the opposite sign**

7. Newton Raphson method has which of the following convergence?

Answer: **Linear convergences**

8.  $x^2 - 3\cos x + xe^x$  is which of the following equation?

Answer: **Transcendental equation**

9. If  $f(x)$  is continuous in the interval  $(a, b)$  and  $f(a)$  and  $f(b)$  are opposite signs, the equation is  $f(x) = 0$  will have positive root is

Answer: **At least one real**

10. A linear equation in one variable has

Answer: **Only one solution**

## UNIT II

1. In Newton Raphson method, the error at any stage is proportional to

ANSWER: **Square of the error .**

2. If  $f(x) = x^3 - 3x^2 + x + 1 = 0$  then a root lies between

ANSWER: **1.5 and 2**

3. The another name of iteration method is

ANSWER: **Method of successive approximation**

4.  $f(x) = e^{-x} - \sin^2 x$  is a

ANSWER: **Transcendental equation**

5.  $E - \Delta =$

ANSWER: **1**

6.  $\Delta^n x^{(n)} =$

ANSWER: **n!**

7.  $\nabla y_n =$  \_\_\_\_\_

ANSWER:  $y_n - y_{n-1}$

8.  $e^{hD} =$

ANSWER:  $1 + \Delta$

9.  $E =$  \_\_\_\_\_

ANSWER:  $1 + \Delta$

10.  $\Delta^n f(x) =$  \_\_\_\_\_

ANSWER:  $a_0 n! h^n$

## UNIT III

1. Backward substitution method is applied in

ANSWER: **Gauss Elimination method**

2. Every homogeneous system of linear equation is consistent and this solution is called \_\_\_\_

ANSWER: **Trivial solution**

3. The number 0.0009875 when rounded off to three significant digits

ANSWER: **0.000988**

4. If a polynomial of degree  $n$  has more than  $n$  zeros, then the polynomial is

ANSWER: **Quadratic**

5. The process of finding the values inside the interval( $X_0, X_n$ ) is called  
ANSWER: **Interpolation**

6. Newton forward interpolation used for  
ANSWER: **Equal Intervals**

7. Taylor series method will be very useful to give some \_\_\_\_\_ for powerful numerical methods.

ANSWER: **Middle value**

8. Polynomials are the most commonly used functions  
ANSWER: **Evaluate, differentiate and integrate**

9.  $\Delta - \nabla =$   
ANSWER:  $\Delta \nabla$

10.  $\nabla y_n =$  \_\_\_\_\_  
ANSWER:  $y_n - y_{n-1}$

#### **UNIT IV**

1.  $\Delta(af(x) + b\phi(x)) =$  \_\_\_\_\_  
ANSWER:  $a\Delta f(x) - b\Delta \phi(x)$

2. The order of the error of Simpson 1/3 rule is higher than that of Trapezoidal rule.

ANSWER: **True**

3. Match the following:

- |                   |   |
|-------------------|---|
| A. Newton-Raphson | 1. Integration                            |
| B. Runge-kutta    | 2. Root finding                           |
| C. Gauss-seidel   | 3. Ordinary Diferential Equations         |
| D. Simpson's Rule | 4. Solution of system of Linear Equations |

ANSWER: **2341**

4. Which of the following formulas is a particular case of Runge-Kutta formula of the second order?

ANSWER: **Euler's modified**

5. The order of error s the Simpson's rule for numerical integration with a step size h is

ANSWER:  **$h^4$**

6. In case of Newton Backward Interpolation Formula which equation is correct to find u?

ANSWER:  **$x - x_n = uh$**

7. Simpson's 1/3rd rule is used only when \_\_\_\_\_

ANSWER: **n is even**

8. While evaluating the definite integral by Trapezoidal rule, the accuracy can be increased by taking \_\_\_\_

ANSWER: **Large number of sub-intervals**

9. In application of Simpson's 1/3rd rule, the interval h for closer approximation should be \_\_\_\_\_

ANSWER: **Even**

10. While applying Simpson's 3/8 rule the number of sub intervals should be \_\_\_\_\_

ANSWER: **multiple of 3**

## UNIT V

1. The numerical solution of a first order differential equation will give a result is

ANSWER: **Value of x and y**

2. A series for y in terms of x, from which the values of y can be obtained by

ANSWER: **Direct substitution**

3. Euler algorithm formula is also be written as

ANSWER:  **$Y(x+y)=y(x)+hf(x,y)$**

4. Euler's improved formula is

ANSWER:  **$Y_{x+1}=y_x+\frac{1}{2}h\{f(x_m, y_m)+f[x_m+h, y_m+hf(x_m, y_m)]\}$**

5. Euler's modified formula is

ANSWER:  **$Y_{x+1}=y_x+h[f(x_m+\frac{1}{2}, y_m+\frac{1}{2}f(x_m, y_m))]$**

6. Third order Runge - Kutta method

ANSWER:  **$\Delta y=\frac{1}{6}(k_1+4k_2+k_3)$**

7. Fourth order Runge – Kutta method

Answer:  **$\Delta y=\frac{1}{6}(k_1+2k_2+2k_3+k_4)$**

8. By Trapezoidal rule  $\int_0^6 \frac{1}{1+x} dx$  by dividing the range into six equal parts is=

Answer: **2.0214**

9.  $\frac{1}{h^2}[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12} \nabla^4 y_n + \dots]$  = \_\_\_\_\_

$$\left(\frac{d^2y}{dx^2}\right)x = x_n$$

**10**  $\int_a^b ydx = \frac{h}{3} [y_1 + 4(y_2 + y_4 + \dots + y_{2n}) + 2(y_3 + y_5 + \dots + y_{2n-1}) + y_{2n+1}]$  is

Answer: **Simpson's  $\frac{1}{3}$ rd rule**

## Numerical Techniques

### K3 Level Questions

#### UNIT I

1. Find a real root of the equations  $x^3+x^2-100=0$ .
2. Find the negative roots of the equations  $x^3-2x+5=0$ .
3. Determine the root of  $xe^x-3=0$  correct to 3 decimal places using the method of false position.
4. Solve  $x^3-x-1=0$  by using bisection method.
5. Solve  $x^3-x^2-2=0$  by regular falsi method.
6. Find the root of  $xe^x-2=0$  which lies between 0 and 1 to 4 decimal places by using falsi position method.
7. Find by Newton Raphson method the real root of  $3x-\cos x-1=0$ .
8. Find by Newton's method correct to 2 decimal places the negative root of the equation  $x^3-21x+35=0$ .
9.  $2x-3\sin x-5=0$  correct to 6 decimal places by Newton's method
10.  $\cos x=3x-2$  solve by using the method of Iteration.

#### UNIT II

1. In the table below, estimate the missing value

|   |   |   |   |   |    |
|---|---|---|---|---|----|
| x | 0 | 1 | 2 | 3 | 4  |
| y | 1 | 2 | 4 | - | 16 |

Explain why its differs from  $2^3=8$ .

2. The following data gives the melting point of an alloy of lead and zinc, where t is the temperature in deg-c and p is the percentage of lead in the alloy.

|   |     |     |     |     |     |     |
|---|-----|-----|-----|-----|-----|-----|
| p | 40  | 50  | 60  | 70  | 80  | 90  |
| t | 184 | 204 | 226 | 250 | 276 | 304 |

3. Using Newton's forward interpolation formula find the value of y when  $x=21$  from the following tabulated values of the functions.

|   |    |    |    |    |
|---|----|----|----|----|
| x | 20 | 23 | 26 | 29 |
|---|----|----|----|----|

|   |        |        |        |        |
|---|--------|--------|--------|--------|
| y | 0.3420 | 0.3907 | 0.4384 | 0.4848 |
|---|--------|--------|--------|--------|

4. Using Newton's formula, find the value of  $f(1.5)$  from the following data.

|      |       |       |       |       |       |
|------|-------|-------|-------|-------|-------|
| x    | 0     | 1     | 2     | 3     | 4     |
| f(x) | 858.3 | 869.6 | 880.9 | 892.3 | 903.6 |

5. Using Newton's formula, find y when  $x=27$  from the following data.

|   |      |      |      |      |      |
|---|------|------|------|------|------|
| x | 10   | 15   | 20   | 25   | 30   |
| y | 35.4 | 32.2 | 29.1 | 26.6 | 23.1 |

6. Using the polynomial of the third degree complete the record given below of the export of a certain commodity during 5 years.

|        |      |      |      |      |      |
|--------|------|------|------|------|------|
| Years  | 1917 | 1918 | 1919 | 1920 | 1921 |
| Export | 443  | 384  | -    | 397  | 467  |

7. Find the missing value from the following data

|   |     |     |      |   |      |
|---|-----|-----|------|---|------|
| x | 2   | 4   | 6    | 8 | 10   |
| Y | 5.6 | 8.6 | 13.9 | - | 35.6 |

8. Find the missing value in the sequence

|       |      |   |       |      |
|-------|------|---|-------|------|
| 15.75 | 17.9 | - | 22.75 | 43.2 |
|-------|------|---|-------|------|

9. Derive the Equation For Newton's forward interpolation

10. Derive the Methodology of Newton Raphson method

### UNIT III

1. Derive Stirling's formula.
2. Derive the maxima and minima of the tabulated function.
3. Find  $dy/dx$  and  $d^2y/dx^2$  at  $x=1.15$  from the table of values of x and y

|   |        |         |         |         |         |         |         |
|---|--------|---------|---------|---------|---------|---------|---------|
| x | 1.00   | 1.05    | 1.10    | 1.15    | 1.20    | 1.25    | 1.30    |
| y | 1.0000 | 1.02470 | 1.04881 | 1.07238 | 1.09544 | 1.11803 | 1.14017 |

4. Find the first two derivatives of  $\sqrt[3]{x}$  at  $x=50$  from the table:

|               |        |        |        |        |        |        |        |
|---------------|--------|--------|--------|--------|--------|--------|--------|
| x             | 50     | 51     | 52     | 53     | 54     | 55     | 56     |
| $\sqrt[3]{x}$ | 3.6840 | 3.7084 | 3.7325 | 3.7563 | 3.7798 | 3.8030 | 3.8259 |

5. Find the values of  $\cos 1.74$  using the values given in the table below:

|       |        |        |        |        |        |
|-------|--------|--------|--------|--------|--------|
| x     | 1.70   | 1.74   | 1.78   | 1.82   | 1.86   |
| Sin x | 0.9916 | 0.9857 | 0.9781 | 0.9691 | 0.9584 |

6. Using the following data, find  $f'(5)$

|      |   |    |    |     |     |     |
|------|---|----|----|-----|-----|-----|
| x    | 0 | 2  | 3  | 4   | 7   | 9   |
| F(x) | 4 | 26 | 58 | 112 | 466 | 922 |

7. From the values in the table given below, find the values of  $\sec 31^\circ$  using Numerical differentiation

|                     |        |        |        |        |
|---------------------|--------|--------|--------|--------|
| $\theta$ in degrees | 31     | 32     | 33     | 34     |
| Tan $\theta$        | 0.6008 | 0.6249 | 0.6494 | 0.6745 |

Hint:  $d/d(\tan \theta) = \sec^2 \theta$

8. In the following data gives corresponding values of pressure and specific volume of a superheated steam.

|   |     |      |      |      |    |
|---|-----|------|------|------|----|
| v | 2   | 4    | 6    | 8    | 10 |
| p | 105 | 42.7 | 25.3 | 16.7 | 13 |

- a) Find the rate of change of pressure with respect to volume when  $v=2$ .

9. The following indicate the velocity 'v' of a body during a time 't' specified. Find its acceleration when  $t=1.1$

|   |      |      |      |      |      |
|---|------|------|------|------|------|
| t | 1.0  | 1.1  | 1.2  | 1.3  | 1.4  |
| v | 43.1 | 47.7 | 52.1 | 56.4 | 60.8 |

10. The population of a certain town is shown in the following table

|       |      |      |      |      |      |
|-------|------|------|------|------|------|
| Years | 1931 | 1941 | 1951 | 1961 | 1971 |
|-------|------|------|------|------|------|

|                           |       |       |       |        |        |
|---------------------------|-------|-------|-------|--------|--------|
| Population in<br>thousand | 40.62 | 60.80 | 79.95 | 103.56 | 132.65 |
|---------------------------|-------|-------|-------|--------|--------|

Find the rate of growth of the population in 1961.

#### UNIT IV

1. Write the notes of Simpson's rule

2. Derive Romberg's method

3. Write the notes of trapezoidal rule.

4. Using the trapezoidal rule, evaluate  $\int_0^2 dx$  from the following table:

|   |      |      |      |      |      |      |       |       |
|---|------|------|------|------|------|------|-------|-------|
| X | 0.6  | 0.8  | 1.0  | 1.2  | 1.4  | 1.6  | 1.8   | 2.0   |
| Y | 1.23 | 1.53 | 2.03 | 4.32 | 6.25 | 8.36 | 10.23 | 12.45 |

5. Use the trapezoidal rule to evaluate the integral of  $y(x)$  from 0 to  $\frac{\pi}{2}$  from the data below.

|   |        |                  |                   |                   |                   |                   |                   |
|---|--------|------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| X | 0      | $\frac{\pi}{12}$ | $2\frac{\pi}{12}$ | $3\frac{\pi}{12}$ | $4\frac{\pi}{12}$ | $5\frac{\pi}{12}$ | $6\frac{\pi}{12}$ |
| Y | .00000 | .25882           | .50000            | .70711            | .86603            | .96593            | 1.00000           |

6. Use the trapezoidal rule to evaluate  $\int_0^1 \frac{x}{x} dx$  dividing the interval into 5 equal parts

7. Using a Simpson's rule, evaluate  $\int_0^{\frac{\pi}{2}} \sin 3x dx$  from the following data:

|          |   |                 |                 |                  |       |
|----------|---|-----------------|-----------------|------------------|-------|
| X        | 0 | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ |
| $\sin x$ | 0 | .7071           | 1.000           | .7071            | 0     |

8. The following table gives the values of  $f(x)$  at equal interval of  $x$

|      |       |       |       |       |       |
|------|-------|-------|-------|-------|-------|
| X    | 0     | 0.5   | 1.0   | 1.5   | 2.0   |
| F(x) | 0.399 | 0.352 | 0.242 | 0.129 | 0.054 |

Evaluate  $\int_0^2 x dx$  using Simpson's rule.

9. Find an approximate value of  $\log_e 5$  by calculating to 4 decimal places by Simpson's rule the integral  $\frac{x}{x}$  dividing the range into 10 equal parts.

10. Numerically integrate  $e^{-x}$  between  $0 \leq x \leq 1$  in steps of 0.1 by each of the following (a) trapezoidal rule

## UNIT V

1. Define Runge-kutta method.

2. Using euler's improved method, find the value of y when  $x=0.1$  given that  $y(0)=1$  and  $y' = x^2 + y$

3. Using  $dy/dx = x^2 + y$ ,  $y(0)=1$ , determine  $y(0.02)$ ,  $y(0.04)$  and  $y(0.06)$  using euler's modified method.

4. Using euler's method and its modified form, obtain  $y(0.2)$ ,  $y(0.4)$ ,  $y(0.6)$  correct to three decimal places if y satisfies.

$$Dy/dx = y - x^2, y(0)=1$$

5. Using modified method of Euler: solve  $dy/dx = 1 - y$ ,  $y(0)=0$  in the range  $0 \leq x \leq 0.3$  taking  $h=0.1$

6. Derive the improved euler's method and the modified euler's method from the Runge-kutta method of second order.

7. Prove that the solution for the equation  $dy/dx = y$ ,  $y(0)=1$  yields  $y_m = [1 + h + 1/2h^2]^2$ , using second order Runge-kutta method.

8. Evaluate the solution at  $x=0.1, 0.2, 0.3$  of the following problem by second order Runge-kutta method.

$$Y' = 1/2(1+x)y^2, y(0)=1$$

9. Tabulated by Runge-kutta method the numerical solution of

$$dy/dx = 1 + y^2 \text{ with } y(0)=0$$

And the step size  $h=0.2$  for  $x=0, 0.2, 0.4, 0.6, 0.8, 0.10$ .

10. Given  $d^2y/dx^2 - y^3 = 0$ ,  $y(0)=10$ ,  $y'(0)=5$  evaluate  $y(0.1)$  using Runge-kutta method.

## Numerical Techniques

### K4 Level Questions

#### UNIT I

1. Find the roots of the equation  $x^3-4x-9=0$  correct to three decimal places by using bisection method
2. Find the real roots of the equation  $p-p^3/3+p^5/10-p^7/42+p^9/216-\dots=0.4431135$
3. compute the real roots of  $x\log_{10}x-1.2=0$  correct five decimal places (regular falsi method).
4. Derive Bisection method.
5. Using Newton-Raphson method establish the formula  $X_{n+1}=1/2(X_n+N/X_n)$  to calculate the square root of N. Hence find the square root of 5 correct to 4 decimal places.

#### UNIT II

1. State and prove Newton's forward interpolation formula.
2. State and prove Newton's backward interpolation formula.
3. The following are data from the steam table

|                                    |       |       |       |       |        |
|------------------------------------|-------|-------|-------|-------|--------|
| Temperature<br>(°C)                | 140   | 150   | 160   | 170   | 180    |
| Pressure<br>(kgf/cm <sup>2</sup> ) | 3.685 | 4.854 | 6.302 | 8.076 | 10.225 |

Using Newton's formula find the pressure of the steam for a temperature of 142°.

4. Estimate  $\exp(1.85)$  from the following table

|        |       |       |       |       |       |       |       |
|--------|-------|-------|-------|-------|-------|-------|-------|
| x      | 1.7   | 1.8   | 1.9   | 2.0   | 2.1   | 2.2   | 2.3   |
| Exp(x) | 5.474 | 6.050 | 6.686 | 7.389 | 8.166 | 9.025 | 9.974 |

5. Given  $\mu_0=-4$ ,  $\mu_1=-2$ ,  $\mu_4=220$ ,  $\mu_5=546$ ,  $\mu_6=1148$  find  $\mu_2$  and  $\mu_3$ .

#### UNIT III

1. State and prove Newton's forward difference formula.
2. State and prove Newton's backward difference formula.
3. From the following table of values of x and y, find  $dy/dx$  and  $d^2y/dx^2$  for  $x=1.05$

|   |         |         |         |         |         |         |         |
|---|---------|---------|---------|---------|---------|---------|---------|
| x | 1.00    | 1.05    | 1.10    | 1.15    | 1.20    | 1.25    | 1.30    |
| y | 1.00000 | 1.02470 | 1.04881 | 1.07238 | 1.09544 | 1.11803 | 1.14017 |

4. The following table gives the results of an observation.  $\theta$  is the observed temperature in degrees centigrade of a vessel of cooling water;  $t$  is the time in minutes from the beginning of observation:

|          |      |      |      |      |      |
|----------|------|------|------|------|------|
| T        | 1    | 3    | 5    | 7    | 9    |
| $\theta$ | 85.3 | 74.5 | 67.0 | 60.5 | 54.3 |

Find approximately the rate of cooling when  $t=8$  using Newton's backward formula.

5. From the following table, find the value of  $x$  for which  $y$  is minimum and find this value of  $y$

|   |       |       |       |       |
|---|-------|-------|-------|-------|
| X | .60   | .65   | .70   | .75   |
| Y | .6221 | .6155 | .6188 | .6170 |

#### UNIT IV

- Dividing the range into 10 equal parts, find the approximate value of  $\int^{\pi} \sin x \, dx$  by (a) Trapezoidal rule, (b) Simpson's rule
- Use Romberg's method to compute  $\int \frac{1}{x^2} dx$  correct to 4 decimal places. Hence deduce an approximate value of  $\frac{1}{e}$ .
- Define Simpson's rule.
- Evaluate  $\int_0^1 \frac{1}{x^2} dx$  correct to three decimal places by trapezoidal rule with  $h=.5, .25, .125$ . Use Romberg's integration to get an accurate value for the definite integral. Hence find the value of  $\log_e 2$ .
- Apply Simpson's rule to evaluate  $\int_0^1 \frac{1}{x^2} dx$  two decimal places, by dividing the range into 4 equal parts.

#### UNIT V

- Derive improved Euler's method.
- Derive second order Runge-kutta method.
- Solve the equation  $dy/dx=1-y$  with the initial condition  $x=0, y=0$ , using euler's algorithm and tabulated the solutions by euler's improved method and euler's modified method. Also compare with the exact solution.
- Apply the fourth order Runge-kutta method, to find an approximate value of  $y$  when  $x=0.2$ , given that  $y'=x+y, y(0)=1$ .
- Solve the system of differential equations  $dy/dx=xz+1, dz/dx=-xy$  for  $x=0.3(0.3)0.9$