REAL ANALYSIS II

MULTIPLE CHOICE QUESTIONS

UNIT 1:

1. The function f is continuous at a € M if $\lim_{x\to a} f(x)$= --------
   (a) f(b)
   (b) f(c)
   (c) f(a) *
   (d) f(x)
2. The open ball of radius r about a is defined by ________
   (a) B[r,a]
   (b) B[a,r] *
   (c) B[-r,a]
   (d) B[-a,r]
3. $\{X_n\}_{n=1}^{\infty}$ is a sequence in M_1 such that $\lim_{n\to\infty} X_n$ = __________
   (a) -a
   (b) o
   (c) 1
   (d) a *
4. Every function from Rd is continuous on __________
   (a) Rd *
   (b) -Rd
   (c) R^2d
   (d) R^3d
5. $\langle M,\rho\rangle$, both M and ϕ are ________
   (a) 1
   (b) Open sets *
   (c) 0
   (d) -1
6. Every subset of Rd is _______
   (a) open
   (b) closed
   (c) Open *
   (d) finite
7. Every open subset G of R ' can be written as ________
   (a) 5
   (b) 3
   (c) open & closed
   (d) G= U In *

8. E is a closed subset of M if E =_________
   (a) E *
   (b) W
   (c) G
   (d) U

9. The set $\overline{E}$ of all limit points of E is called the _______
   (a) open
   (b) Closure of E *
   (c) open
   (d) compact

10. The subset A of M is said to be dense in M if $\overline{A}$ =_______
   (a) N
   (b) L
   (c) M *
   (d) U

UNIT 2:

1. Every bounded subset of R^2 is _________
   (a) Bounded
   (b) Not bounded
   (c) Totally bounded *
   (d) None

2. Every subsequence of a convergence sequence is ___________
   (a) Divergent
   (b) Continuous
   (c) convergent *
   (d) Both (a) and (b)

3. Class of functions are called __________
   (a) contractions *
   (b) Distractions
   (c) Divergent
   (d) Convergent

4. $\{X_{n}k\}_{k=1}^{\infty}$ is a Cauchy subsequence of _________

(a) ∞

$$\{X_n\}_{n=1}^{\infty}$$

(b) $\{Xn\}_{n=1}^{\infty}$ *

(c) 0

(d) Xn

5. The Metric space <M, ρ> is both complete and totally bounded is said to be ________
   (a) scalar
   (b) complete
   (c) compact *
   (d) discrete

6. The space Rd with finite subset is __________
   (a) discrete
   (b) complete
   (c) compact *
   (d) scalar

7. If M is a compact metric space then M has a ___________
   (a) Heine Borel Property *
   (b) vector
   (c) scalar
   (d) mean value theorem

8. If F1, F2,......,Fn €f 1 then F1∩F2∩.......∩Fn ≠ __________
   (a) 1
   (b) 0
   (c) 2
   (d) ϕ*

9. The real valued function f as continuous at the point a€R' if given ε>0 there exist δ>0 such that __________
   (a) 0
   (b) 1
   (c) $|f(x)-f(a)|<\varepsilon$ *
   (d) ϕ

10. If the real valued function f is continuous on the closed bounded interval [a,b], then f is _____________
   (a) Uniformly Continuous *
   (b) continuous
   (c) convergent
   (d) divergent

UNIT 3:

1. If χ is a characteristic function of rational numbers [0,1] then for any interval JC [0,1] then $m[\chi,J]$=____________

(a) 0 *
(b) ∞
(c) $-\infty$
(d) 1

2. If f is bounded function on the closed bounded interval [a,b], we say that f is_________

(a) Riemann – integral *
(b) Continuous
(c) Bounded
(d) None

3. $\int_{\bar{a}}^{b} f(x).dx = \int_{a}^{\bar{b}} f(x).dx$=___________

(a) 1 *
(b) 0
(c) ∞
(d) $-\infty$

4. $\int_{a}^{\underline{b}} f(x).dx$=___________

(a) lub $U(f,\sigma)$
(b) glb $U(f,\sigma)$ *
(c) lub $U(g,\sigma)$
(d) glb $U(g,\sigma)$

5. $\bigcup_{m=1}^{\infty} E_m$=___________

(a) E *
(b) M
(c) En
(d) 0

6. E*m U E**m=_________

(a) E *
(b) E*
(c) ∩E*
(d) UE**

7. U[f,τ] – L[f,τ]<__________

(a) d|J|
(b) a|J| *
(c) b|J|
(d) c|J|

8. $\int_a^c f + \int_c^b f$ =____________

(a) $\int_b^a f$
(b) $\int_a^b f$ *
(c) $\int_c^d f$
(d) $\int_d^c f$

9.If f ϵ R(a,b) g ϵ R(a,b) and (f+g) ϵ R (a,b) then $\int_a (f+g)$=____________

(a) $\int_a^b f + \int_a^b g$ *
(b) $\int_a^b g + \int_a^b f$
(c) $\int_a^b f$
(d) $\int_a^b g$

10. Show that f(x) = 2x+1 is integrating on [1,2] and ∫ (2x+1).dx=__________

(a) 0
(b) 2
(c) 4 *
(d) -4

UNIT 4:

1. g is continous function defined by g(x)=________

(a) < cos x , sin x > *
(b) < -cos x , sin x >
(c) <cos x, -sin x>
(d) <-cos x, -sin x>

2. f is uniformly continous on R1, if given ε > 0 , there exists _______

(a) δ>o*
(b) δ>1
(c) δ<1
(d) δ<0

3. Subset B of R^2 consisting of graph of y=sin(1/x) , o<x≤1 together with the ____interval on y-axis from <0,-1> to<0,1>

(a) Open
(b) Closed*
(c) Continuous
(d) Both (a) and (b)

4. Subset of R^2 is bounded, iff it is contained in some square whose edge has ______length

(a) infinite
(b) finite*
(c) none
(d) both (a) and (c)

5. Subset of R^2 is bounded ,iff it is contained in some ______ whose edge has finite length

(a) Square*
(b) rectangle
(c) triangle
(d) none

6. The interval _________is not a bounded subset of R′

(a) (0,∞)*
(b) (∞,0)
(c) (0,1)
(d) (1,∞)

7. Bounded and totally bounded are not at all _________

(a) non- equivalent
(b) Equivalent*
(c) finite
(d) none

8. Since ρ(ej,ek)=√2 ,if jǂk , the sequence ρ1,ρ2,........... has no________

(a) Cauchy’s sequence
(b) Cauchy subsequence*
(c) Convergent sequence
(d) Divergent sequence

9. Metric space[0,1] is _______for [0,1] is a closed subset of R′

(a) Compact
(b) Connect
(c) Complete*
(d) None

10. Metric space is denoted by _________

(a) <M,p>
(b) <M,ρ>*
(c) <m,ρ>
(d) <m,p>

UNIT5:

1. If f has a derivative at c and it is denoted by __________

(a) f(c)
(b) f ′(c) *
(c) f(1)
(d) f′(0)

2. Define Rolle’s theorem.
3. Write down the statement of mean value theorem.
4. If f has a derivative at c then it is _________ at c.

(a) Neither or nor continuous
(b) Bounded
(c) Continous *
(d) Both (a) and (b)

5. If f has derivative at c and g has derivative at f(c) then g_o f has a _________at c.

(a) Compact
(b) Complete
(c) Connectedness
(d) Derivative *

6. If E is any subset of a metric space M then __________

(a) Ē C E
(b) E C Ē *
(c) E Ɔ Ē
(d) Ē Ɔ E

7. The union of a infinite number of closed sets need not be a __________

(a) Closed set *
(b) Open set
(c) Both (a) and (b)
(d) Union

8. If A and ϕ are both open and closed in metric space < A, ρ> then A is said to be _______

(a) Complete
(b) Compact
(c) Connected *
(d) Closed

9. If a subset A of the metric space <M, ρ> is totally bounded then A is _________

(a) Unbounded
(b) Bounded *
(c) Continuous
(d) Closed

10. The space R′ is complete but not __________

(a) Connect
(b) Continuous
(c) Compact*
(d) None

K2 QUESTIONS:

UNIT 1:

1. If a homeomorphism from M_1 onto M_2 exist , we say that M_1 and M_2 are
   Answer: Homeomorphism
2. Metric space M is totally bounded if it has __________________ sets.
   Answer: finite number
3. If f is continuous at a, then ω[f;a] = _______
   Answer: 0
4. If f is not continuous at a, then ω[f;a] > _______
   Answer: 0
5. Let A = [0,1], which of the following subsets of A are open subset of A.
   Answer: (1/2,1)
6. Ā1∩A2 = ____, A1∩Ā2= ϕ
   Answer: ϕ
7. x and f are both continuous then x° f is _______
   Answer: Continuous *
8. A is not bounded we write diam A = _________
   Answer: ∞ *
9. If T: M→m is a contraction on M then ρ (t_x,t_y) ≤ ________
   Answer: α (ρ(x, y))
10. If every Cauchy sequence of sequence if points in M converges to points in M is called a ___________
    Answer: Complete metric space

UNIT 2:

1. Function f is bounded if its range f(A) is a __________
   Answer: Bounded subset
2. If f is a real valued function on a set A that f attains a maximum value of a€A if _________
   Answer: f(a) ≥ f(x), x€A
3. If f is a real valued function on a set A that f attains a minimum value of a€A if __________
   Answer: f(a)≤f(x), x€A
4. A function is continuous if and only if it is uniformly continuous then it is said to be __________
   Answer: Compact metric space
5. The subset E of R′ is said to be________.

   Answer: Measure zero

6. U   En = __________
n=1

Answer: Measure zero

7. I 1 =[X∘,X1], I 2 =[X1,X2].........In=[Xn-1,Xn] are called________

Answer: Component interval of σ

8. U[f:σ] ≥ _____________
Answer: L(f,σ)

9. ∫ f(x).dx = __________
a

Answer: lub L(f,σ)

10. If χ is a characteristic function of relational numbers [0,1] then for any interval J C [0,1] then M[χ,J]=___________

Answer: 1

UNIT 3:

1. If T* is any refinement of T , it may be show that L[f;T] ≤ ________________

Answer: L [f ; T *]

2. ∑ |In| converges to__________________
n

∞
Answer: U En
n=1

b
3. ∫ λ f =________

b
Answer: λ ∫
a

4. The function f is defined by f(x)= x^2+ 2x , 0<x<4, then f(x) is =_________

Answer: 15

5. F1 and F2 are closed subsets of metric M, then F1UF2 is ___________

Answer: Closed

6. The union of an infinite number of closed sets is __________

Answer: need not be a closed set

7. The set is open if and only if its complement is___________

Answer: Closed

8. If f is continuous at a iff_____________

Answer: $\lim_{n\to\infty} X_n = a$

9. M is a metric space with________ property

(a) Heine borel

10.If |x-a| < δ, then the limit exceeds from_________

Answer: $-\infty<a<\infty$

UNIT 4:

1. Metric space <M,ρ> the sets M and ϕ are both _________
   Answer: Open and closed
2. If A is not bounded, then we write diam A equal to_______
   Answer: ∞
3. $\bigcap_{n=1}^{\infty} F_n$ contains_________

   Answer: One point
4. The non-empty subsets A1,A2,……An of M exits such that__________

   Answer: diam Ak < 1

5. Which one is correct form of 'contradiction'______

Answer: $\rho(tx,ty) \leq \alpha(\rho(x,y))$

6. Choose the correct example for conuity of the inverse function

Answer: f(x)=x

7. g is continuous function defined by______

Answer: g(x)= < cosx,sinx>

8. Let g be the continous function defined by g(x)=<cosx,sinx> , $0 \leq x \leq 2\pi$, then g^{-1} is____________

Answer: Continuous

9. f is homomorphism of _______

Answer: m1 onto m2

10. The space Rd with ∞ subset cannot be ______

Answer: Compact

UNIT 5:

1. If A is a closed subset of a compact metric space <M, ρ> then A is also ________

Answer: Compact

2. If M is a compact metric space then M has a ________ property.

Answer: **Heine – Borel**

3. If the metric space M has a Heine-Borel property then M is ________

Answer: Compact

4. If f has a derivative at c then it is ________ at c.

Answer: Continous

5. If f has derivative at c and g has derivative at f(c) then $g \circ f$ has a ________at c.

Answer: Derivative

6. If E is any subset of a metric space M then _________

Answer: E C Ē

7. The union of a infinite number of closed sets need not be a _________

Answer: Closed set

8. If A and ϕ are both open and closed in metric space < A, ρ> then A is said to be _______

Answer: Connected

9. If a subset A of the metric space <M, ρ> is totally bounded then A is _________

Answer: Bounded

10. The space R′ is complete but not __________

Answer: Compact

REAL ANALSIS II

K2 QUESTIONS:

Unit 1

1. If the real valued functions f and g are continuous at a ϵ R', then so are f+g, f-g and fg. If g(a)≠0, then f/g is also continuous at 'a'.
2. If f and g are real valued functions, if f is continuous at a, and if g continuous at f(a), then g ° f is continuous at 'a'.
3. The real valued function f is continuous at a ϵ R' iff given ϵ>0 there exist δ>0 such that |f(x)-f(a)| <ϵ, |x-a|<δ.
4. The real valued function f is continuous at a ϵ R' iff the inverse image under f of any open ball B[f(a), r] about f(a) contains a open ball B[a, δ] about a.
5. The real valued function f is continuous at a ϵ R', iff whenever Xn_{n}^{∞} is the sequence of real numbers convergent to 'a'. Then the sequence Xn_{n}^{∞} converges to f(a) ie) f is continuous at 'a' iff $\lim_{n\to\infty} Xn$ =a
   ⇨ $\lim_{n\to\infty} Xn$ =f(a)
6. The function f is continuous at a ϵ M if any one of the following condition holds
   (i) Given ε>0 there exist δ>0 such that $\rho_2(f(x),f(a))<\varepsilon$, $\rho_1(x, a)<\delta$
   (ii) The inverse image under f of any open ball B[f(a),ε] about f(a) contains an open ball B[a, δ] about 'a'.
   (iii) Whenever Xn_{n}^{∞} is a sequence of points in M, converging to 'a'.

   Then the sequence Xn_{n}^{∞} of points in M_2 converges to f(a).

7. Let <M_1, ρ_1>, <M_2, ρ_2> be metric spaces and let $f:M_1\to M_2$ and $g:M_2\to M_3$. If f is continuous at A ϵ M_1 and g is continuous at f(a) ϵ M_2. Then g(f) is continuous at A.
8. If f and g are continuous function from a metric space M_1 into a metric space M_2 then so are f+ g , f. g and f g further more g(x) ≠0, x ϵ M_1.
9. Let £ be a non-empty family of open subsets of a metric space M. Then U_G is also in open subset of M.
10. If G_1 and G_2 are open subsets of metric space M then $G_1 \cap G_2$ is also an open set.

Unit 2:

1. Let <M, ρ> be a metric space and let 'A' be a proper subset of M then the subset G of A is an open subset of metric space <A, ρ> iff there exists an open subset G_M of metric space <M, ρ> such that $G_A = A \cap G_{M}$ (ie) A set is open in metric space <A, ρ> iff it is intersection of a set with 'A' that is open in metric space <M, ρ>.
2. Let <M, ρ> be a metric space and Let A be a subset of M, then if 'a' has either one of the following properties it has the other.
   i) Non empty subset A_1 and A_2 of M such that $A=A_1 \cup A_2$, $A'\cap A_2=\varphi$, $A_1\cap A_2'=\varphi$
   ii) When <A, ρ> metric space then there is no set except A and φ which is both open and closed in metric space <A ,e> .This we say that A is connected.

3. The subset of A of R_1 is connected iff whenever $a \epsilon A$, $b \epsilon A$ with $a<b$ then $C \epsilon A$ for any C, such that $a<c<b$ that is whenever $a \epsilon A$, $b \epsilon A$, $a<b$, then (a, b)C A.
4. Let F be a continuous function from metric space M_1. If M_2 is connected then the range of F into connected.
5. Let M be a metric space then M is connected iff every continuous characteristic function on M, is constant c (ie) M is connected iff the function identically ‘zero’ and the function identically ‘1’ are the only characteristic functions on M that are continuous on M.
6. If A_1 and A_2 be connected subsets of a metric space M and if $A_1 \cap A_2 \neq \varphi$ then $A_1 \cup A_2$ is also connected.
7. If the subset A of the metric space $<M , \rho>$ is totally bounded then A is bounded.
8. The subset A of the metric space $<M, \rho>$ is totally bounded if and only if for every $\varepsilon>0$, A Ͻ a finite subset $\{x_1,\ldots..,x_n\}$ which is ε dense in A.
9. Let $<M, \rho>$ be a metric space , the subset A of M is totally bounded iff every sequence of points of A contains a Cauchy sub sequences.

Unit 3:

1. The metric space $<M, \rho>$ is compact iff every sequence of points in M has a subsequence converging to a point in M
2. If A is a closed subset of the compact metric space $<M, \rho>$ then the metric space $<A, \rho>$ is also compact
3. Let A be a subset of a metric space $<M, \rho>$ is $<A, \rho>$ is compact , then A is also closed subset of $<M, \rho>$
4. If M is a compact metric space then M has a Heine Borel property
5. If the metric space M has a Heine Borel property then M is compact.
6. The metric space M is compact iff whenever f is a family of closed subset of M with the finite intersection property then $n F \neq \varphi$ $F \epsilon f_1$
7. Let f be a continuous function from compact metric space M_1, into the metric space M_2^1 , then the range of f Ie) $f(m_1)$ is also compact
8. Let f be a continuous function from the compact metric space $M_1 \rightarrow M_2$ then the range of $f(M_1)$ of f is a bounded subset of M_2
9. If the real valued function f is continuous on closed bounded interval in R’ then f must be bounded
10. If the real valued function f is continuous on the compact metric space M then f attains a maximum value at some point of M also f attains a maximum value at some point of M.

Unit 4:

1. If each of the subset $E_1, E_2,\ldots.$ Of R’ is of measure zero , then $\cup_n^\infty$ E_n is also of measure zero
2. If f be a bounded function on [a, b] then every upper sum for f is greater than or equal to every lower sum of f that is if σ and T are any two subdivisions of [a, b] then $u[f, \sigma] \geq L[f,T]$

3. Let f be a bounded function on the closed bounded interval [a, b] then f ϵ R [a, b] if and only if f is continuous at almost every point in [a, b]
4. If ω [f; x]< a for each x in a closed bounded interval J then there is a subdivision τ(J) such that U[f, τ]-L[f, τ] < a |J|
5. If f ϵ R[a, b] and a<c<b then f ϵ R [a, c], f ϵ R[c, b] and , = +
6. If f ϵ R[a, b] and λ is any real numbers then λ ϵ R [a, b] and =λ
7. Every countable subset of R' as measure zero
8. If f ϵR(a, b) and λ is any real number then λ ϵ R(a, b) and =λ
9. State and prove chain rule.
10. STATE AND

Unit 5:

1. Mean value theorem or Lagrange's mean value theorem.
2. If f is a continuous real valued function on the interval J and if f'(x) >0 for all x in J except possibly the end point of J then F is strictly increasing on J
3. Let f and g be continuous functions on the closed bounded interval [a, b] with g(a) f(b)if both f and g has derivative at each point of (a, b) and f'(t) and g'(t)are not both equal to zero for any c ϵ (a, b) then there exist a point c ϵ (a, b) such that f'(c)/g'(c)= f(b)-f(a)/g(b)-g(a)
4. If f is a continuous on a closed bounded interval [a, b] and if F(x)= at,

   a≤ x≤ b

   Then F(x)=f(x), a≤ x≤ b
5. If the real valued function f has the derivative at the point c ϵ R' then f is continuous at c.
6. If f ϵ R [a, b] if f(x) = dt a≤ ≤ and if f is continuous at $x_{\circ}$ ϵ [a, b] then f'(x)=f($x_{\circ}$)
7. Let f be a continuous real valued function on the closed bounded interval [a, b]. If the maximum value for f is attained at c where a<c<b and if f'(c) exists then f'(c) =0
8. Let f be a continuous real valued function on the closed bounded interval [a, b] . If the minimum value of f is attained at c where a<c<b and if f'(c) exists then f'(c) =0
9. If f'(x)=0, for every x in the closed bounded interval [a, b] then f is constant and closed interval [a, b] f(x)=c, a≤ b≤ c for some c ϵ R
10. IF f'(x)=g'(x) for all x in the closed bounded interval [a, b] when f-g is constant i.e) f(x)=g(x)+c.

REAL ANALSIS II

K3 QUESTIONS:

UNIT 1:

1. Every open set G of R' can be written G= U In where I_1, I_2,…… are mutually disjoint open intervals.
2. Let <M_1,ρ_1> and <M_2,ρ_2> be a metric spaces and let f:M_1→M_2 then f is continuous on M if f^{-1}(G) is open in M_1 whenever G is open in M_2. That is f is continuous iff the inverse image of every of every open set is open.
3. Let E be a subset of metric space M, then the point x ϵ M is a limit point of E iff every open ball B[x ; r] about x contains atleast one point of E.
4. If F_1 and F_2 are closed subsets of metric M, then F_1 U F_2 is also closed.
5. If £ is a family of closed subsets of a metric space M then intersection or ∩F is a closed set.

UNIT 2:

1. If <M, ρ> is a complete metric space, A is closed subset of M then <A, ρ> is also complete.
2. Let <M, ρ> be a complete metric space for n ϵ I . Let Fn be a closed bounded subset of M such that
   (i) F1 Ɔ F2 Ɔ……Fn Ɔ Fn+1 Ɔ……
   (ii) Diam Fn→0 as n→∞ then $\cap_{n}^{\infty}$ Fn contains presely one point.
3. Let <M, ρ> be a complete metric space if T is a contraction on M then , there is one and only one point x in one such that T x =x
4. The subset A of Rd is totally bounded iff it contains only a finite number of points
5. If F is a continuous real valued function on the closed bounded interval [a,b] then f takes on every value between f(a) and f(b)

UNIT 3:

1. If the real valued function f is continuous on the closed bounded interval [a, b] then f attains the maximum or minimum values at point [a, b]
2. If f is a one to one continuous function from the compact metric space m1 onto the metric space m2, then f^{-1} is continuous on m2 and hence f is homomorphism of m1 onto m2
3. Let <M,ρ_1> be a compact metric space if f is continuous function from M_1 into M_2 a metric space <M_2,ρ_2> then f is uniformly continuous on m1
4. If the real valued function is continuous on the closed bounded interval [a, b] then f is uniformly continuous on [a, b]

5. Let <M_1,ρ_1> be a metric space and let A be a dense subset of M_1. If f is a uniformly continuous from <A, ρ_1> into a complete metric space <M_2,ρ_2> then f can be extended to a uniformly continuous function f from M_1into M_2

UNIT 4:

1. If f ϵ R(a, b) g ϵ R(a, b)and (f+g) ϵ R(a, b) then ∫ +∫ ∫
2. Properties of the Rie-mann integral

3. If f ϵ R[a, b], g ϵ R[a, b] and if f(x)≤ g(x) almost everywhere (a≤ x≤ b) then ∫ ≤ ∫

4. If f ϵ R[a, b] then |f| ϵ R[a, b] and |∫ |≤ ∫ | |

5. Let f be a bounded function on the closed bounded interval [a, b] then f ϵ R[a, b] if and only if for each ε>0 there exist a subdivision σ([a, b]) such that U(f,σ)<L(f,σ)+ε

UNIT 5:

1. Second fundamental theorem of calculus
2. If f and g both have derivatives at c ϵ R' then f+g, f-g,fg also have derivatives at c ϵ R' and (f+g)'c=f'(c)+g'(c)and (f-g)'c=f'(c)-g'(c) and (fg)'c=f'(c).g(c)+f(c) g'(c) further more if g'(c)≠0, then f/g has a derivatives at c and (f/g)'c=g(c)f'(c)-f(c)g'(c)/$[g(c)]^2$
3. Suppose f has a derivative of c and g has a derivative at f(c), then φ=g°f has a derivative at c and φ'(c)=g'(f(c))f'(c)
4. Let φ be a real valued function on closed bounded interval [a, b] such that φ' is continuous on [a, b]. Let A=φ(a) B=φ(b) then if f is continuous on φ[a, b] we have ∫ dx=∫ φ'(x) dx
5. If f has a derivative at every point of [a, b] . Then f' takes an every value between f'(a) and f'(b)