REAL ANALYSIS I

K1-LEVEL QUESTIONS

UNIT I

1. Set is a collection of well defined ----------objects.
   a. **Distinct**
   b. same
   c. Equal
   d. None
2. The objects in a set are called its ------------.
   a. objects
   b. Elements
   c. both a and d are true
   d. None
3. In the set A ={a,b,c,d}, which statement is true?
   a. e belongs to A
   b. c belongs to A
   c. a belongs to A
   d. both b and c are true
4. As a set which has no element is called ------------
   a. infinite set
   b. Empty set
   c. universal set
   d. None
5. The Cartesian product of A and B is the set of all -----------------where $a \in A$ and $b \in B$.
   a. triplet
   b. single element
   c. Ordered pairs
   d. None
6. In a function definition, y = f(x), y is called -----------
   a. Image of x under f
   b. range
   c. domain
   d. None
7. In a function f from A into B, the set A is called --------of f.
   a. range

b. element

c. object

d. Domain

8. Composition of functions satisfy --------------law.

a. Associative

b. identity

c. absorption

d. None

9. If f(x) = f(y) implies x = y, then the function is called ----------------

a. One to One

b. Many to one

c. onto

d. None

10. If there exists one to one correspondence between the sets A and B, then A and B are called -------------

a. Equivalent

b. not equivalent

c. neither a nor b

d. either a or b

Unit II

11. In sequence s_i (i=1,2,.....) is called ----------- .

a. i^{th} term of the sequence.

b. n^{th} term of the sequence.

c. j^{th} term of the sequence.

d. none

12. Which one is the sequence of the subsequence {2,3,5,7,.......}.

a. {1,2,3,.........}

b. {0,1,2,3,..............}

c. {2,3,5,7,11,...........}

d. none

13. {1,3,5,......} is the subsequence of -------------.

a. {1,2,3,.........}

b. {0,1,2,3,..............}

c. {2,3,5,7,11,...........}

d. none

14. Which one is the subsequence of {1,0,1,0,.....}?

a. {1,1,1,1,......}

b. {0,0,0,0,......}

c. both a and b

d. neither a nor b

15.The sequence $\{S_n\}$ has the limit L, then ----------.

a. **$|S_n – L| < \mathcal{E}$**

b. $|S_n + L| < \mathcal{E}$

c. $|S_n\ \ L| < \mathcal{E}$

d. None

16.In {1, ½, 1/3, ...} , then the limit of the sequence is --------------

a. 0

b. 1

c. 2

d. None

17.If a sequence of nonnegative numbers, then its limit------------

a. $L \leq 0$

b. $L \geq 0$

c. $L < 0$

d. None

18.If the sequence of real numbers has the limit L, then we say that the sequence is ------------.

a. Convergent

b. Divergent

c. Oscillatory

d. None

19.A sequence c an not converge to ------- limit.

a. More than one

b. More than two

c. same

d. None

20.Which one is convergent sequence?

a. {+1, - 1, +1, - 1, ...}

b. {1,2,3,........}

c. {1,1,1,1.......}

d. None

Unit III

21.The limit of the sum of two convergent sequence is --------.

a. L + M

b. L – M

c. LM

d. L/M

22.If $\lim_{n\to\infty} \quad _{n}$ then $\lim_{n\to\infty} \quad _{n}$?

a. cL

b. L

c. both a and b

d. None

23.If 0 <x <1, then $^{n}\,{}^{\infty}_{n}$ converges to ---------.

a. 1

b. 0

c. – 1

d. None

24.The limit of the difference of two convergent sequence is ------------.

a. L + M

b. L – M

c. LM

d. L/M

25.The limit of the product of two convergent sequence is ---------.

a. L + M

b. L – M

c. LM

d. L/M

26.The limit of the quotient of two convergent sequence is --------.

a. L + M

b. L – M

c. LM

d. L/M

27.$\lim_{n\to\infty} \frac{\ _{n} \quad _{n}}{\ _{n} \quad _{n}} = ?$

a. 1

b. 2

c. 0

d. None

28.If two sequences are diverge to infinity then their sum = ?

a. Converge to L

b. Diverge to infinity

c. both a and b

d. None

29.If two sequences are diverge to infinity then their product = ?

a. Converge to L

b. Diverge to infinity

c. Oscillatory

d. None

30.If $_{n}{}_{n}^{\infty}$ diverges to infinity and $_{n}{}_{n}^{\infty}$ converges, then their sum is --

a. Diverge to infinity

b. Convergent

c. oscillatory

d. None

Unit IV

31.--------------- is an ordered pair $\langle\ _{n}{}_{n}^{\infty}\ ,\ _{n}{}_{n}^{\infty}\ \rangle$.

a. The series

b. The sequence

c. $\sum_{n}^{\infty}\ _{n}$

d. both a and c

32.In series, $_{n}\ \cdots\ _{n},\ \in$, the number $_{n}$ is called--------

a. The j^th term

b. The i^th term

c. The n^th term

d. None

33.The number ----------- is called the n^th partial sum of the series.

a. $_{n}$

b.

c. $_{n}$

d. None

34.If n is odd, what is the n^th partial sum of the series 1 – 1 + ... $-1\ ^{n}$ $\cdots$?

a. 1

b. – 1

c. 0

d. None

35.If n is even, what is the n^th partial sum of the series 1 – 1 + ... $-1\ ^{n}$ $\cdots$?

a. 1

b. 0

c. – 1

d. None

36.If the sequence is converges to A then the corresponding series is ---------

a. Converges to B

b.Diverges

c. Converges to A

d. either a or b

37.If the sequence is diverges then the corresponding series is ----------.

a. Diverges

b. Converges

c. either a or b

d. neither a or b

38.The series $\sum_{n} \quad 1/$ is ------------.

a. Convergent

b. Divergent

c. Both a and b

d. either a or b

39.Which one is alternate series ?

a. $\sum_{n} \quad -1 \; ^{n} \; _{n}$

b. $\sum_{n} \quad -1 \; ^{n} \; _{n}$

c. Both a and b

d. Either a or b

40. If $\sum_{n} \quad | \; _{n}|$ converges the we say that $\sum_{n} \quad _{n}$ --------------.

a. Converges conditionally

b. Converges absolutely

c. Both a and b

d. Either a or b

Unit V

41.In metric spaces, f be a real valued function whose domain includes all points in some open interval -------------- except possibly the point a itself.

a. (a – h, a + h)

b. (a + h, a + h)

c. (a – h, a – h)

d. None

42.lim $_{\to}$, then the number is called -------.

a. Right hand limit of f at a

b. Left hand limit of f at a

c. both a and b

d. none

43.lim $\to$, then the number is called -------.

a. Left hand limit of f at a

b. limit of f at a

c. Right hand limit of f at a

d. None

44.If both the left and right hand limit exists and equal to L, then it is called ----------------.

a. Limit of f.

b. Limit of a

c. Limit of b

d. none

45.If lim $\to$ and lim $\to$ then lim $\to$?

a. L

b. F

c. A

d. None

46.What is the condition for the real valued function f is nondecreasing on J?

a. $f(x) \geq f(y)$ whenever $x>y$; $x,y \in J$

b. $f(x) \leq f(y)$ whenever $x<y$; $x,y \in J$

c. $f(x) \leq f(y)$ whenever $x=y$; $x,y \in J$

d.$f(x) \geq f(y)$ whenever $x<y$; $x,y \in J$

47.What is the condition for the real valued function f is nonincreasing on J?

a. $f(x) \geq f(y)$ whenever $x<y$; $x,y \in J$

b. $f(x) \leq f(y)$ whenever $x<y$; $x,y \in J$

c. $f(x) \leq f(y)$ whenever $x=y$; $x,y \in J$

d. $f(x) \leq f(y)$ whenever $x > y$; $x,y \in J$

48.What is the condition for the real valued function f is monotone on J?

a. $f(x)\leq f(y)$ whenever $x<y$; $x,y \in J$ and $f(x) \geq f(y)$ whenever $x<y$; $x,y \in J$

b. $f(x)\leq f(y)$ whenever $x>y$; $x,y \in J$ and $f(x) \geq f(y)$ whenever $x>y$; $x,y \in J$

c. $f(x)\geq f(y)$ whenever $x<y$; $x,y \in J$ and $f(x) \leq f(y)$ whenever $x<y$; $x,y \in J$

d. None

49.If the left hand limit exists then we say that f is ------------ on (a,b).

a. Bounded above
b. Bounded below
c. Bounded
d. None

50. If the right hand limit exists then we say that f is ------------ on (a,b).
a. Bounded above
b. Bounded below
c. Bounded
d. None

K2-LEVEL QUESTIONS

UNIT I

1. A set A is said to be --------------- if A is equivalent to the set I of positive integers.
   Countable
2. ------------ is the another name for the Countable set.
   Denumerable
3. A set which is not countable is called --------------
   Uncountable
4. The set of all integers is ----------
   countable
5. The set of all ------------ is uncountable.
   Real numbers
6. The Countable union of countable set is ------------.
   Countable
7. The set of all ---------- is countable.
   Rational numbers
8. The glb of (7,8) is ----------.
   7
9. The lub of (7, 8) is -------------.
   8
10. If A is any nonempty subset of R that is bounded above, then A has a -----
    LUB

Unit II

11. Write one example for divergent sequence?
    {+1, - 1, +1, - 1, ...}
12. If the sequence of real numbers convergent to L, then it's any subsequence is convergent to --------.
    L
13. -------------- of a convergent sequence of real numbers converge to the same limit.
    All subsequences

14. If the limit of a sequence is infinity, then the sequence is -------------.

Divergent

15. In divergent sequence, the limit of the sequence approaches to -------------

Infinity or minus infinity

16. Write one example for a divergent sequence?

$\boldsymbol{n} \quad {}^{\infty}_{\boldsymbol{n}}$

17. Which subsequence of the sequence {1, -2, 3, -4, 5, -6,......} is divergent to minus infinity?

{-2,-4, -6,.....} and {1,3,5,.....}

18. The sequence neither diverge to minus infinity nor infinity then the sequence is called --------

Oscillates

19. If the sequence real numbers is convergent then it is ---------.

Bounded

20. ------------ is a sequence which is either nonincreasing or nondecreasing or both.

Monotone sequence

Unit III

21. What is the sum of {0,1,0,2,0,3,.......} and {1,0,2,0,3,0,........}?

{1,1,2,2,3,3,......}

22. The sum of {1,0,1,0,1,0,......} and {0,1,0,1,0,1,.....} is a --------------- sequence.

Convergent

23. The limit superior of the sequence {1, -1, 1, -2, 1,-3, 1, -4,......} is ---------

1

24. In a convergent sequence, limit superior of the sequence is equal to -------

Limit of the sequence

25. In --------- sequence Limit superior of the sequence is equal to Limit of the sequence.

Convergent

26. $\lim_{\to\infty} inf \ -1 \quad = ?$

$-$

27. In a convergent sequence limit inferior of the sequence is equal to ---------

a. Limit of the sequence

28. In --------- sequence Limit inferior of the sequence is equal to Limit of the sequence.

Convergent

29. If the sequence of real numbers converges then that sequence is a ---------

Cauchy sequence

30. Every Cauchy sequence is a --------------- sequence.

Convergent.

Unit IV

31. If $\sum \quad_1$ converges but $\sum \quad_1 | \quad |$ diverges then we say that $\sum \quad_1$ -------------.

Converges conditionally

32. Which one is conditionally convergent series?

1 – ½ + 1/3 – ¼ +...

33. If we rearranging an --------series then the series remains the same.

Absolutely convergent

34. If we rearranging an --------series then the rearrangement of that series is converges.

Conditionally convergent

35. If a series converges absolutely then its rearrangement is -------.

Absolutely convergent

36. If $\sum \quad_1$ is dominated by $\sum \quad_1$ then------------.

$|\boldsymbol{a}|_{\boldsymbol{n}} \leq | \quad |_{\boldsymbol{n}}$

37. If $\sum \quad_1$ is dominated by $\sum \quad_1$ and $\sum \quad_1$ converges absolutely then ------- converges absolutely.

$\sum_{\boldsymbol{n}} \quad \boldsymbol{a}_{\boldsymbol{n}}$

38. If $\sum \quad_1$ is dominated by $\sum \quad_1$ and $\sum \quad_1$ converges absolutely then $\sum \quad_1$ converges absolutely. This test is called ----------------.

Comparison test

39. ------------ test is used to test the convergence of the series $\overline{| \; |}$.

Root test

40. If |x| <1 then $\sum \quad_1 n^{10,000}$ is ------------.

Converges absolutely

Unit V

41. What is the condition for the real valued function f is strictly increasing on J?

f(x) <f(y) whenever x<y ; x,y ∈ J

42. What is the condition for the real valued function f is strictly decreasing on J?

f(x) >f(y) whenever x<y ; x,y ∈ J

43. In a metric space <M,ρ>, ρ is called -------- on M.

Metric

44. From the given below, which one is a metric space?

a. |x – y|

b. x – y

45. If ρ is a metric for a set M then 2ρ is ---------------.

Metric

46. If ρ and σ are two metrics for a set M, then σ + ρ is ---------.

Metric

47. In a metric space, a convergent sequence of points of M, then the corresponding sequence is a --------------- sequence.

Cauchy

48. A sequence of points in any metric space cannot converge to ------- distinct limits.

Two

49. In some metric spaces there are -------------- sequences which are not convergent.

Cauchy

50. A sequence converges in <M, ρ> if and only if it converges in <M, σ> and the limits are same then the two metrics are----------------.

Equivalent

K3- LEVEL QUESTIONS

UNIT 1

1. If f : A→B and if X C B, Y C B, then $f^{-1}(X\cap Y)=f^{-1}(X)\cap f^{-1}(Y)$. In words, the inverse image of the

2. If f : A→B and if X C B, Y C B, then $f^{-1}(XUY)=f^{-1}(X)\ U\ f^{-1}(Y)$. In words, the inverse image of the union of two sets is the union of the inverse images.

3. If $A_1,A_2,.....$ are countable sets, then * U_n^{∞} A_n is countable . In words, the countable union of countable sets is countable.

4. If f :A→B , g : B→C , h : C→D , prove that h◦ (g◦f)=(h◦g)◦f.

5. Let f(x)=2x (-∞<x<∞). Can you think of function g and h which satisfy the two equation

   $g\circ f=2gh$, $h\circ f=h^2-g^2$?

6. The set of all rational numbers in [0,1] is countable.

7. If B is an infinite subset of the countable set A , then B is countable.

8. The set of all rational numbers is countable.

9. Which of the following define a 1-1 function?

   (a) $f(x)= e^x(-\infty<x<\infty)$,

   (b) f(x)= (-∞<x<∞),

   (c) f(x)=ax+b(-∞<x<∞),a,b ϵ R.

10. If A is any nonempty subset of R that is bounded below, then A has a greatest lower bound in R.

UNIT 2

1. If $Sn\ {}_n^{\infty}$ is a sequence of non negative numbers and if $lim_{n\to\infty}$ Sn =L , then L ≥0

2. If the sequence of real numbers $Sn\,{}_{n}^{\infty}$ is convergent to L, then $Sn\,{}_{n}^{\infty}$ cannot also converge to a limit distinct from L. That is, if $lim_{n\to\infty}$ Sn =L and $lim_{n\to\infty}$ Sn=M, then L=M.

3. If the sequence of real number $Sn\,{}_{n}^{\infty}$ is convergent to L, then any subsequence of $Sn\,{}_{n}^{\infty}$ is also convergent to L.

4. All subsequence of convergent sequence of real numbers converge to the same element.

5. If the sequence of real numbers $Sn\,{}_{n}^{\infty}$ is convergent, then $Sn\,{}_{n}^{\infty}$ is bounded.

6. A non decreasing sequence which is bounded above is convergent.

7. The sequence $1\ \frac{}{n}\ {}^{n}\ {}_{n}^{\infty}$ is convergent.

8. A non decreasing sequence which is not bounded above diverges to infinity.

9. A non increasing sequence which is bounded below is convergent. A non increasing sequence which is not bounded below diverges to minus infinity.

10. Prove $lim_{n\to\infty}$ 2n/(n+3) =2.

UNIT-3

1. If {$s_n\,{}_{n}^{\infty}$ and {$t^n\,{}_{n}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n$=L, and if $\lim_{n\to\infty}\ {}_n$=M, then $\lim_{n\to\infty}\ S_n$+t_n)=L+M. In words, the limit of the sum (of two convergent sequences) is the sum of the limits.

2. If {$s_n\ {}_n$ is a sequence of real numbers,if c ϵ R, and if $\lim_{n\to\infty} sn$ = L, then $\lim_{n\to\infty}\ sn$= cL.

3. (a) If 0< x < 1, then {$x^n\,{}_{n}^{\infty}$ converges to 0. (b) If 1< x < ∞ then {$x\,{}_{n}^{\infty}$ diverges to infinity.

4. If {$s\,{}_{n}^{\infty}$ and {$t\,{}_{n}^{\infty}$ are sequence of real numbers, if $\log_{n\to\infty} sn$ = L, and if $\log_{n\to\infty}\ n$= M, then $\log_{n\to\infty}\ s_n$-t_n) =L - M.

5. If {$s_n\,{}_{n}^{\infty}$ and {$t_n\,{}_{n}^{\infty}$ are convergent sequence of real numbers, if $s_n \le t_n$ (n ϵ I), and if $\lim_{n\to\infty} s_n$=L, $\lim_{n\to\infty}\ {}_n$=M, then L ≤ M.

6. If $\{s_n{}_{n}^{\infty}$ is a sequence of real numbers which converges to L, then $\{s_n{}_{n}^{\infty}$ converges to L^2.

7. If $\{s_n{}_{n}$ and $\{t_n{}_{n}$ are sequences of real numbers, if $\lim_{n\to}$ s_n=L, and if $\lim_{n\to}$ $_n$=M, then $\lim_{n\to}$ s $_n$

8. If $\{t_n{}_{n}$ is a sequence of real numbers, if $\lim_{n\to}$ $_n$=M where M$\neq$ 0, ▯ n $\lim_{n\to}$ $1/$ $_n$)=1/M.

9. If $\{s_n{}_{n}$ and $\{t_n{}_{n}$ are sequences of real numbers, if $\lim_{n\to}$ s_n=L, and if $\lim_{n\to}$ $_n$=M where where M$\neq$ 0, ▯ n $\lim_{n\to}$ s_n/t_n) =L/M.

10. Prove $\lim_{n\to}$ $\frac{n}{n}$ $\frac{n}{}$=–.

UNIT-4

1. If $\sum_n$ $_n$ is a convergent series, then $\lim_{n\to}$ $_n$=0

2. If $\sum_n$ $_n$ is a series of non negative numbers with $s_n=a_1+...+a_n$ (n I) then (a) $\sum_n$ $_n$ converges if the sequence $\{s_n{}_n$ is bounded; (b) $\sum_n$ $_n$ diverges if $\{s_n{}_n$ is not bounded.

3. If $\sum_n$ $_n$ converges to A and If $\sum_n$ $_n$ converges to B, then $\sum_n$ $_n+b_n$ converges to A+B. Also, if c ϵ R, then $\sum_n$ $_n$ converges to c A.

4. (a) If 0<x<1, then $\sum_n$ ^{n} converges to 1/(1-x). (b) If x$\geq$ 1, then $\sum_n$ ^{n} diverges.

5. The series $\sum_n$ $\frac{-}{n}$ is divergent.

6. If $\sum_n$ $_n$ is a convergent series of positive numbers, then there is a sequence $\{\epsilon_n{}_n$ of positive numbers which converges to zero but for which $\sum_n$ $_n$ $_n$ still diverges.

7. If $\{a_n{}_n$ is a sequence of positive numbers such that

(a) $a_1\geq$ $_2\geq$ .... $\geq$ $_n\geq a_{n+1}\geq\cdots$

(b) $\lim_{n\to}$ $_n$=0, then the alternating series $\sum_n$ -1 ^{n} $_n$ is convergent.

8. If $\sum_n$ $_n$ converges absolutely, then $\sum_n$ $_n$ converges.

9.(a) . If $\sum_n$ $_n$ converges absolutely then both $\sum_n$ $_n$ and $\sum_n$ $_n$ converge. however, (b) . If $\sum_n$ $_n$ converges conditionally, then both $\sum_n$ $_n$ and $\sum_n$ $_n$ diverge.

10. Let $\sum_n$ $_n$ be a conditionally convergent series of real numbers. Then for any x R there is a rearrangement of $\sum_n$ $_n$ which converges to x.

UNIT-5

1. If lim $_{\to}$ =L and lim $_{\to}$ =M, then f(x)+g(x) has a limit as x→a and, in fact, lim $_{\to}$ +g(x)]=L+M.

2. If lim $_{\to}$ =L and lim $_{\to}$ =M, then

(a) lim $_{\to}$ -g(x)]=L-M

(b) lim $_{\to}$ g(x)]=L . M

And if M≠ 0,

(c) lim $_{\to}$ /g(x)]=L/M.

3. Let f be a non decreasing function on the bounded open interval (a,b). If f is bounded above on (a,b), then lim $_{\to}$ exists. Also, if f is bounded below on (a,b) then lim $_{\to}$ exists.

4. Let f be a non increasing function on the bounded open interval (a,b). If f is bounded above on (a,b), then lim $_{\to}$ exists. Also, if f is bounded below on (a,b) then lim $_{\to}$ exists.

5. If f is a monotone function on the open interval (a,b), and if c ϵ (a,b), then lim $_{\to}$ and lim $_{\to}$ both exists.

6. Let <M,ρ> be a metric space and let 'a' be a point in M. Let f and g be real-valued functions whose domains are subsets of M. If † lim $_{\to}$ = L and lim $_{\to}$ =N, then

lim $_{\to}$ +g(x)]=L+N,

lim $_{\to}$ -g(x)]=L-N,

lim $_{\to}$ g(x)]=LN.

And, if N≠ 0

$\lim_{\to}$ /g(x)]=L/N.

7. Let<M,ρ> be a metric space. If {s_n n is a convergent sequence of points of M. Then {s_n n is a Cauchy.

8. Define metric spaces with example.

9. prove that d:R X R→[0,∞) BY

d(x,x)=0 x∈R,

d(x,y)=1, x,y ∈ R ; x≠ is a metric spaces.

10. Show that if ρ is a metric for a set M then so is 2ρ.

K4- LEVEL QUESTIONS

UNIT I

1. Find the glb and lub for the following sets.

   (a) (7,8) (b). [1,2] (c). [1,3) (d). (- 1,1]

2. If A is any nonempty subset of R that is bounded above, then A has a least upper bound in R.
3. If A,B are subset of S, then (A UB)' = A' ∩ B' and (A ∩ B)'= A' U B'
4. Prove, for any sets A, B, C, that (A∩ B) ∩ C=A∩(B∩C)
5. Prove (AUB)-(A∩B)=(A-B)U(B-A)

UNIT II

6. Prove that $\{\sqrt{n}\}_{n}^{\infty}$ diverges to infinity.
7. let $\{s_n\}_{n}^{\infty}$ be the sequence defined by

$S_1=1$

$S_2=2$

$S_{n+1}=s_n+s_{n-1}$ (n=3,4,5,....). Find s_8.

8. Define convergent sequence with example.
9. Explain monotone sequence with example.
10. Explain the bounded sequence with example.

UNIT III

11. If $\{s_n\}_n$ and $\{t_n\}_n$ are sequences of real numbers that diverge to infinity, then so do their sum and product. That is, $\{s_n+t_n\}_n$ and $\{s_n t_n\}_n$ diverge to infinity.
12. If $\{s_n\}_n$ and $\{t_n\}_n$ are sequences of real numbers, If $\{s_n\}_n$ diverges to infinity, and if

$\{t_n\}_n$ *is bounded*, then $\{s_n+t_n\}_n$ diverges to infinity.

13. If $\{s_n\}_n$ diverges to infinity and if $\{t_n\}_n$ converges, then $\{s_n+t_n\}_n$ diverges to infinity.
14. If $\{s_n\}_n$ is a convergent sequence of real numbers, then $\lim_{n\to} \sup s_n = \lim_{n\to} s_n$

15. If {s_n $_n$ is a convergent sequence of real numbers, then
lim inf$_{n\to}$ s_n=lim$_{n\to}$ s_n.

16. If {s_n $_n$ is a Cauchy sequence of real numbers, if only if {s_n $_n$ is a convergent.

UNIT IV

17. If $\sum_n$ $_n$ converges absolutely to A, then any rearrangement $\sum_n b_n$ of $\sum_n$ $_n$ also converges absolutely to A.

18. If the series $\sum_n$ $_n$ and $\sum_n b_n$ converge absolutely to A and B, respectively, then AB =C where C = $\sum_n$ $_n$ and $c_n=\sum^n$ $_k b_{n-k}$ (k=0,1,2,3,...)

19. If $\sum_n$ $_n$ is dominated by $\sum_n b_n$ and $\sum_n$ □ $_n$□=∞, □*en* $\sum_n$ □b_n□=∞.

20. If $\sum_n$ $_n$ is dominated by $\sum_n b_n$ where $\sum_n b_n$ converges absolutely, then $\sum_n$ $_n$ also converges absolutely. Symbolically, if $\sum_n$ $_n$ ≪ $\sum_n b_n$ and $\sum_n$ □b_n□ ∞, □*en* and $\sum_n$ □ $_n$□ < ∞.

21. If lim sup$_{n\to}$ $\overline{\square\ _n\square}$=A then the series of real numbers $\sum_n$ $_n$ (a) converges absolutely if A<1, (b) diverges if A>1

UNIT V

22. Show that if ρ and σ are both metric for a set M, then ρ+σ is also a metric for M.

23. For P < x_1, y_1> and Q<x_2, y_2>, define

σ (P,Q)=□x_1-x_2□+□y_1-y_2□.

Show that σ is a metric for set of ordered pairs of real numbers.

Also, if

Ŧ (P,Q)=max (□x_1-x_2□, □y_1-y_2□),

Show that Ŧ defines a metric for a same set.

24. Show that a sequence of points in any metric spaces cannot converge to two distinct limits.

25. Show that lim $_\to$ $\sqrt{\quad 3}$=2.

26. Prove

lim $_\to$ ——————=1.