# B. Sc. Part II Semester IV
# PHYSICS Paper VIII
# DSC- D2 - WAVES AND OPTICS-II


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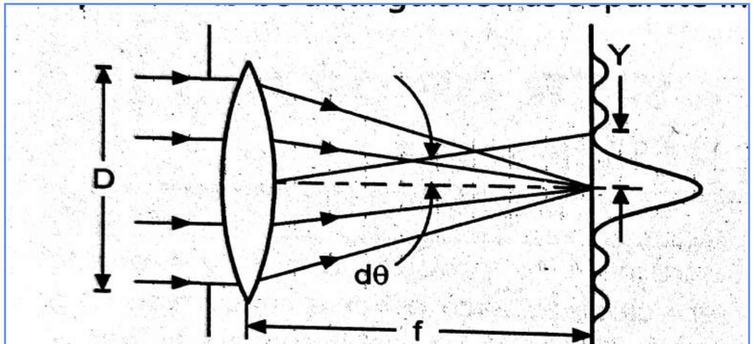
- **Resolving power of optical instruments,**

Resolving power of an optical instrument, is the ability to resolve two objects very close to each other i. e. is the ability to produce separate images of two objects close to each other. Resolving power of an optical instrument (an eye, a microscope, a telescope etc.) depends upon the aperture (diameter of the lens or mirror) and wavelength of light used.

While ability of instrument to show magnified view of objects is called magnifying power. Magnifying power of objective lens depends on its focal length. By proper choice of lens, it is possible to increase the size of the image.

When a beam of light from point source, passes through a slit produces a diffraction pattern, called Fraunhoffer diffraction at single slit. i. e. we observed image of a point source is a Fraunhoffer diffraction (i. e. in which intensity distribution pattern contains central maxima and it’s both sides we see alternate minima and maxima), i.e. in which there is a central bright circle surrounded by alternate dark and bright rings. The first dark ring corresponds to first minima in the intensity distribution.




(a) The image of point object is a diffraction pattern

If there are two-point objects lying close to each other, then two diffraction patterns are produced, which may overlap and it may be difficult to distinguish them as separate. An optical system is able to resolve two closed point objects.

A microscope resolves the linear distance between two close objects. Let $\Delta x$ is the smallest distance resolved, then resolving limit of microscope is $\Delta x$ . A telescope gives geometrical resolution between two (far away) objects. Let $\Delta\theta$ is the smallest distance resolved, then resolving limit of telescope is $\Delta\theta$ . The prism or grating, resolve two nearby spectral lines, let $\Delta\lambda$ is the smallest difference of wavelength resolved, then resolving limit of telescope is $\Delta\lambda$ .

Thus, the minimum distance (smallest separation $\Delta x$, $\Delta\theta$,$\Delta\lambda$) between the two objects (images) or spectral lines is known as limit of resolution. Hence the resolving power of an optical instrument is equal to the reciprocal of limit of resolution.

$$Resolving\ power = \frac{1}{\Delta x}\ or\ \frac{1}{\Delta\theta}$$

For spectral resolution, if $d\lambda$ is the wavelengths to be resolves and $\lambda$ is their mean wavelength, then $\frac{\lambda}{\Delta\lambda}$ is taken as the measure of resolving power of the instrument.

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# •Rayleigh's criterion for the limit of resolution,

We know that the method of separating two objects very close to each other is called resolution. The minimum distance (smallest separation $\Delta x$, $\Delta\theta$,$\Delta\lambda$) between the two objects (images) or spectral lines is known as limit of resolution. The ability or capacity of any optical instrument to just resolve the images of two close object point or spectral lines is called resolving power.

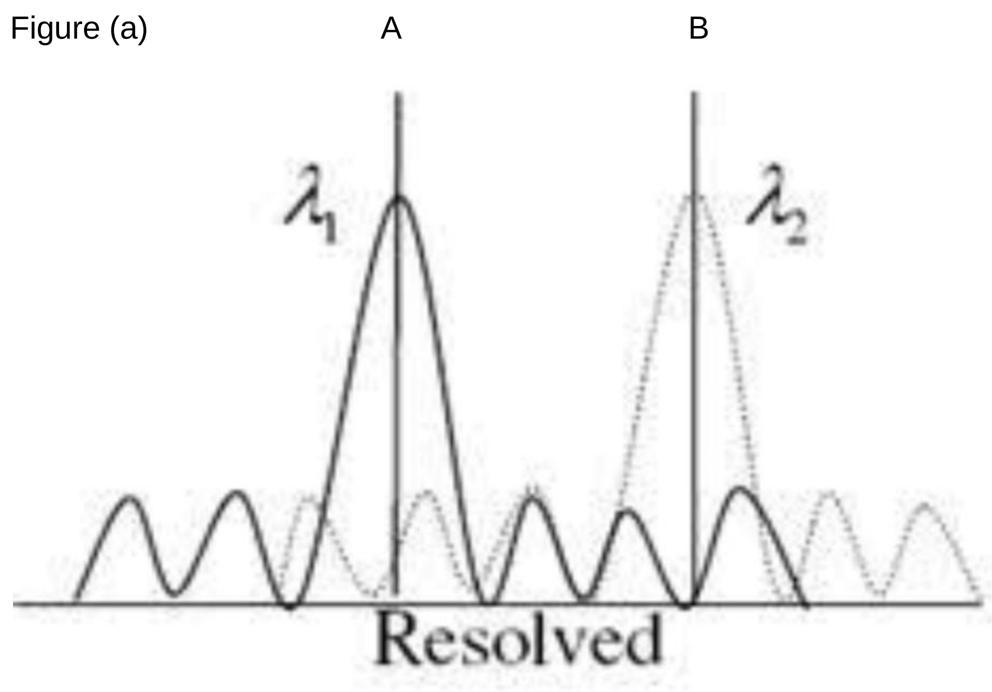
To express limit of resolution and hence resolving power of an optical instrument as in terms of a numerical value, Rayleigh proposed an arbitrary criterion, it is known as Rayleigh's criterion.

**According to Rayleigh's, images of two close objects point are just resolved if principal maximum of one in their diffraction pattern falls on first subsidiary (secondary) minimum of other or at just resolution, the angular separation between the principal maxima is equal to half the angular width of principal maximum of either.**

**(or angular separation of first minima from the central maximum).**

**According to Fraunhoffer diffraction, the position of first minima (from central maxima) on either side is given by condition $a sin\alpha = \lambda$ (condition for just resolution), where $\lambda$ is wavelength of light used $a$ is the aperture width of rectangular slit and $\alpha$ is angular separation of first minima from the central maximum. To explain Rayleigh's criterion for limit of resolution, let us consider Fraunhoffer diffraction patterns due to two neighboring point objects A and B.**

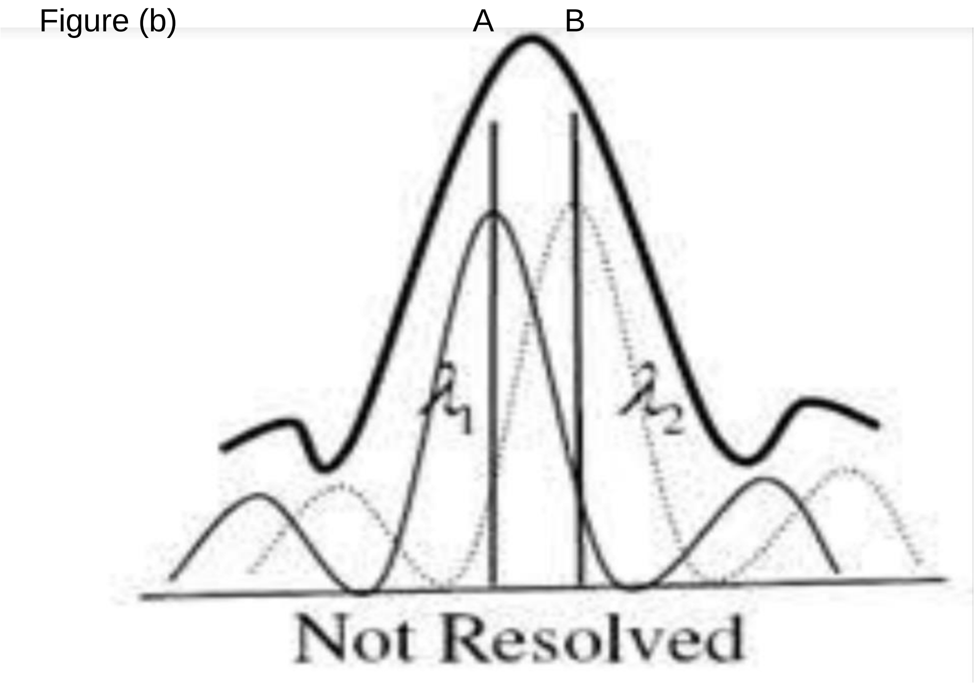
Let us consider figure (a), shows A and B are central maxima of diffraction patterns of two spectral lines of wavelengths $\lambda_1$ and $\lambda_2$.




The difference in angle of diffraction is large and two images can be seen as separate ones. The angle of diffraction corresponding to central maximum of image B is greater than angle of diffraction corresponding to first minimum to right of image A, hence two spectral lines will appear well resolved.

In figure (b), consider two centrals maximum of diffraction patterns A and B having wavelengths $\lambda_1$ and $\lambda_2$.

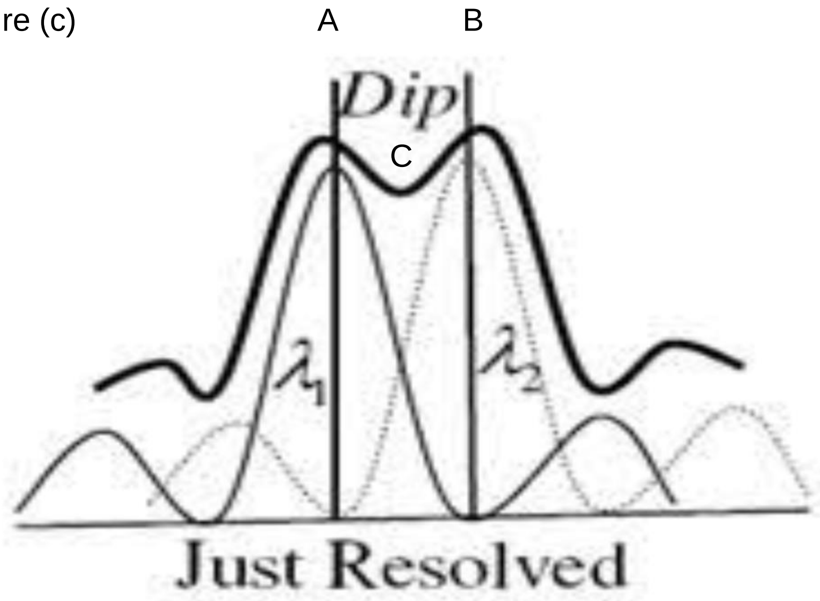
The angle of diffraction corresponding to central maximum of image B is less than angle of diffraction corresponding to first minimum of A. Hence two images overlap and they cannot be distinguished as separate images.




The resultant intensity curve gives a maximum is higher than intensities of A and B. Thus, in this case two spectral lines are not resolved.

**In figure (c), the position of central maximum of A coincides with position of first minimum of B. Similarly position of B coincides with position of first minimum of A.**

Figure (c)




**Then the resultant intensity curve shows a dip at point C i.e. in middle of central maxima of A and B. The intensity at point C is less than that of intensity at point A and B. Therefore, two lines are just resolved. Thus, in this case two spectral lines are not resolved.**

## a) Resolving power of a telescope:

$$d\alpha = \frac{r}{f} \;...............(1)$$

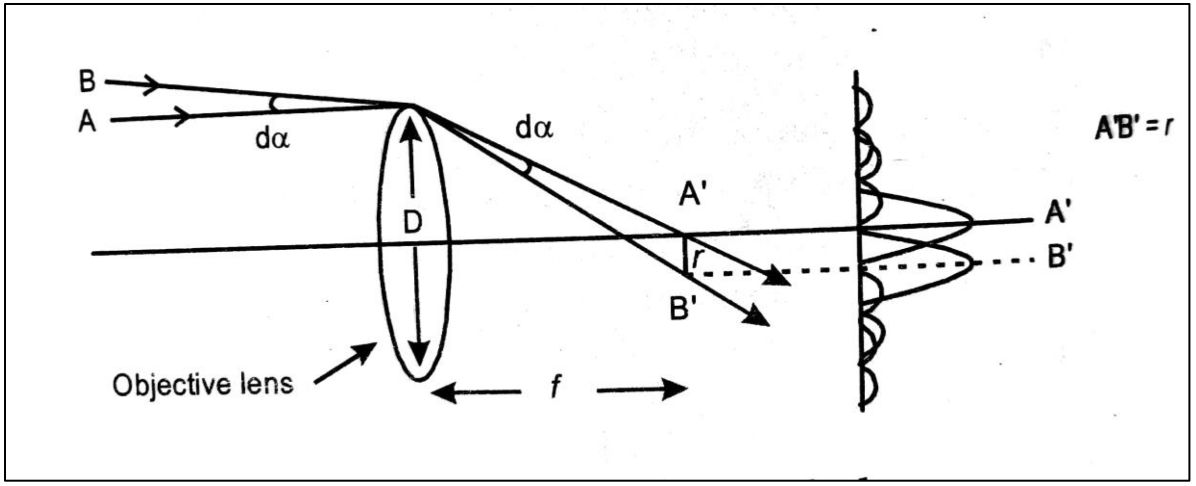
Applying Rayleigh's criterion for just resolution

$D\sin(d\alpha) = \lambda$ ............ for rectangular aperture

But for circular aperture

$$D\sin(d\alpha) = 1.22\lambda$$

$$\sin(d\alpha) = d\alpha = \frac{1.22\lambda}{D} \;..............(2)$$




From (1) and (2) $\frac{r}{f} = \frac{1.22\lambda}{D}$

Therefore, limit of resolution $r = \frac{1.22\lambda f}{D}$

Resolving power of telescope $\frac{1}{r} = \frac{D}{1.22\lambda f}$

## b) Resolving power of a microscope:

$\delta = MA + AN$

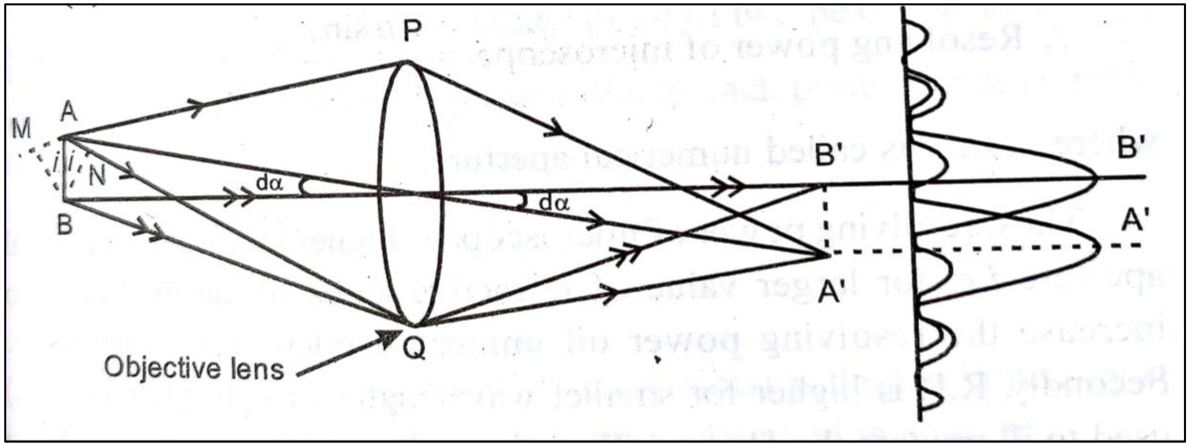
$\delta = h\, sini + h\, sini = 2h\, sini$ ................(1)

Applying Rayleigh's criterion for just resolution

$\delta = Dsin(d\alpha) = \lambda$ ............ for rectangular aperture

But for circular aperture

$\delta = Dsin(d\alpha) = 1.22\lambda$ ...............(2)




From (1) and (2)

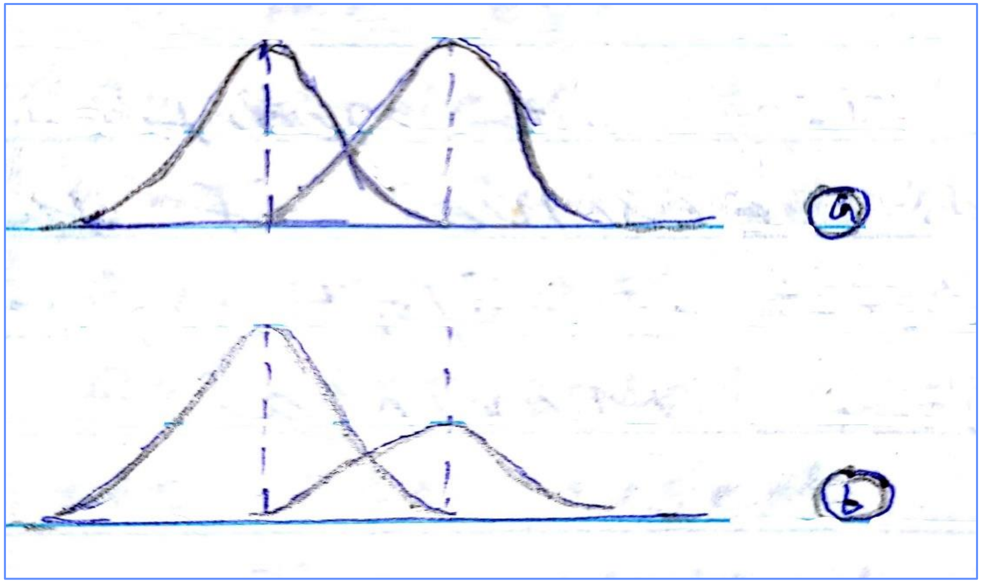
$2h\, sini = 1.22\lambda$

Therefore, limit of resolution $h = \frac{1.22\lambda}{2\, sini} \cong \frac{1.22\lambda_0}{2\mu\, sini}$

Resolving power of telescope $\frac{1}{h} = \frac{2\mu\, sini}{1.22\lambda_0}$

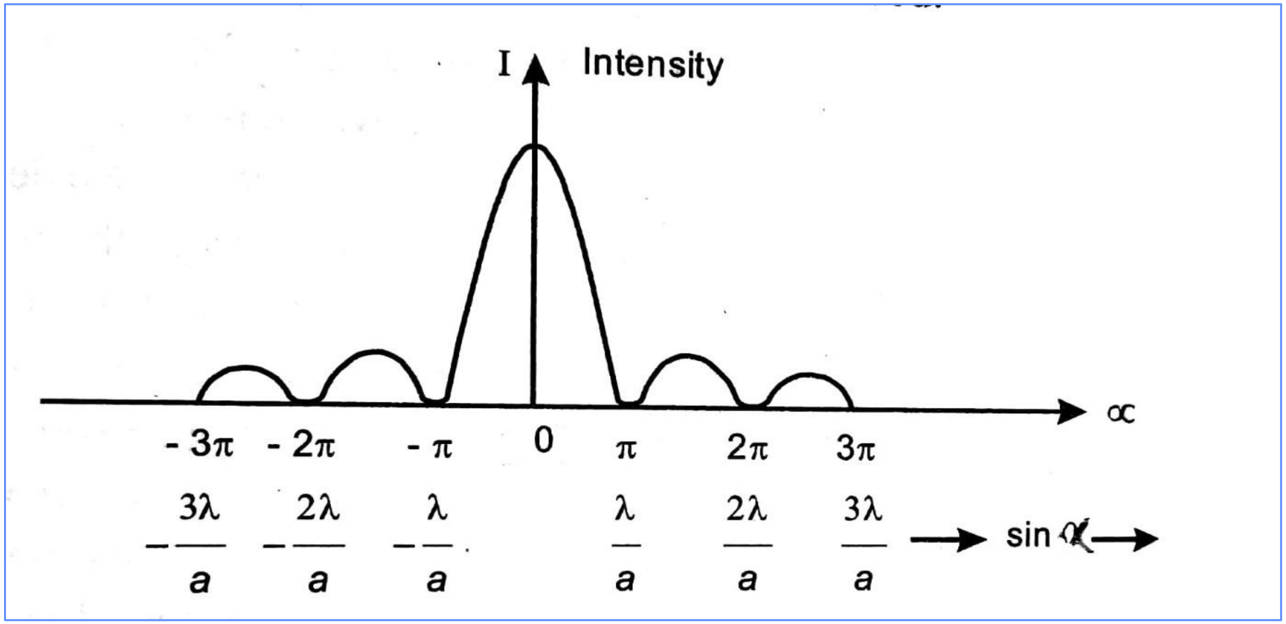
## Modified Rayleigh's criterion,

**Rayleigh's criterion applied only for two images of equal intensity. Consider images of unequal intensity. In figure (a), shows two images of equal intensity and figure (b) shows two images of unequal intensity. In both figure the separation of the centers of the images is equal to half width of one of them.**




**But if intensities of two images are unequal then their resultant intensity distribution does not give information about resolution. So that criterion has to be modified.**

**Let us consider rays from a point object passing through an aperture of width (a), produce Fraunhoffer diffraction pattern as shown in fig. (c) below, in which there is a central maximum at $\alpha = 0$, and minima at $\alpha = n\pi$, where $n = 1,2,3, \dots \dots \dots .$ .**



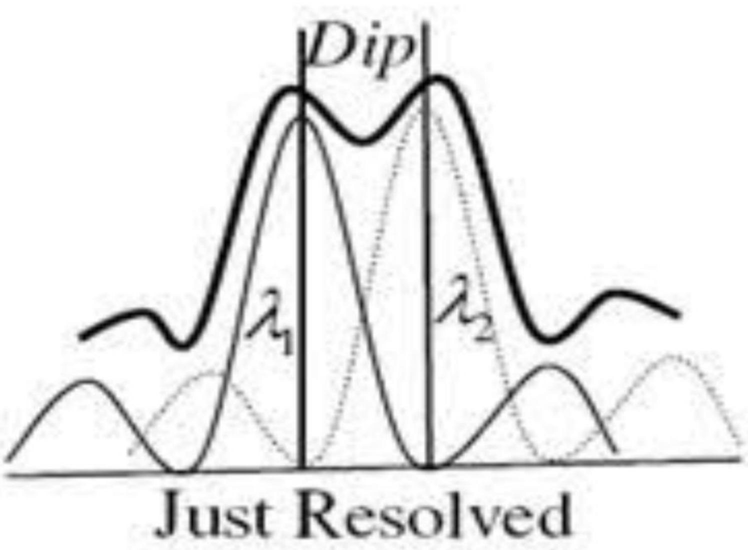

**Hence position of first minimum on either side of central maximum is $a \sin \alpha = \lambda$, where $\lambda$ is wavelength of light. The intensity distribution in Fraunhoffer diffraction pattern is given by**

$I = I_{max} \cdot \left(\frac{sin\alpha}{\alpha}\right)^2$ **; where $I_{max}$ is maximum intensity (central maximum) at $\alpha = 0$**

**Now according to Rayleigh's criterion, two neighbouring images (or objects) are just resolved when central maximum of one pattern just falls at the first minimum of other (fig. (d)). In resultant intensity distribution, the intensities at the two central maxima are $I_{max}$ each, which corresponds to $\alpha = 0$.**

The first minimum occurs at $\alpha = \pi$. Hence two maxima are at an angular separation of $\pi$. At the middle point, the intensity $I_{mid}$ in the diffraction pattern, where $\alpha = \frac{\pi}{2}$ is given by

$$I_{mid} = I_{max} \cdot \left( \frac{sin\frac{\pi}{2}}{\frac{\pi}{2}} \right)^2 = \frac{4}{\pi^2} I_{max}$$



But due to two overlapping diffraction patterns the total intensity at the middle of two peaks where a dip is observed is given by

$$I_{mid} = 2 \times \frac{4}{\pi^2} I_{max} = \frac{8}{\pi^2} I_{max} = 0.81\, I_{max}$$

**Thus we say that, intensity at mid-point (at dip) is 81% of intensity at either maxima. Also angular separation less than** $\frac{\pi}{2}$ **, the two peaks (images) are not resolved.**

**Therefore, Rayleigh's modified criterion for just resolution may be stated as, two neighbouring images are just resolved, when the intensity at the dip point is 0.81 times the intensity at either maxima** $(I_{max})$.